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METHODS FOR MONITORING FRACTIONALLY SAMPLED

MULTIPLE STREAM PROCESSES

by

Jeffrey Wayne Lanning

A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

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ARIZONA STATE UNIVERSITY

August 1998

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ABSTRACT

This study considers multiple stream systems where it is possible to monitor only a fraction of the total streams at a given time. This situation is of interest in those processes where the speed of production is great and includes a large number of streams, but the ability to monitor the process is not fully automated and unable to keep up with the speed of production.

A method for determining the probability of detecting a shift from target of any fraction of the streams (including none of the streams) is presented. In addition to the mathematics involved in computing this detection probability, a computer program is given which automates the process and quickly gives a result for any number of streams allowing an infinite number of combinations of stream shift scenarios to be examined. Results from several of these scenarios are tabulated and graphed.

Adaptive approaches to system monitoring are applied to multiple stream processes in general and the fractional sampling problem specifically. This represents the first application of adaptive techniques to multiple stream processes. The average time to signal for an adaptively-monitored, fractionally-sampled multiple stream process is developed using a Markov chain procedure. The average time-to-signal results are used to identify promising adaptive sampling schemes for monitoring multiple stream processes using fractional samples. The adaptive fraction approach is shown to give superior results to the fixed fraction scheme and often yields satisfactory results compared with those obtained by sampling all the streams involved in a process.

Monitoring the variance of the fractionally sampled stream average is shown to provide protection against situations where only one, or just a few streams shift, rather than all the streams in the system. Finally an in depth example is provided by means of a case study where the methods described in this study are applied.

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BIOGRAPHICAL SKETCH

Captain Jeffrey Wayne Lanning was born in Holland, Michigan, on the 14th of November 1964. He received his elementary education at Childs Elementary School in Bloomington, Indiana. His secondary education was completed at Laramie Senior High School in Laramie, Wyoming in 1983. After high school he attended the University of Wyoming for one year before being accepted to the U.S. Air Force Academy. He graduated with military distinction from the Air Force Academy in June of 1988 holding a Bachelor of Science degree and received a regular commission in the U.S. Air Force. His first assignment with the Air Force involved three years on a classified test program where he served as the lead human factors analyst. In March of 1993 Captain Lanning completed a Master of Science degree in operations research at the Air Force Institute of Technology. Following graduation he moved on to the Headquarters Air Force Operational Test and Evaluation Center for two years as an effectiveness analyst and program manager and became the Center's focal point for operational testing of the global positioning system. In August 1995 Captain Lanning entered the Graduate College at Arizona State University to pursue a doctorate in Industrial Engineering with an emphasis on quality and reliability engineering. Following graduation Captain Lanning will take a position as a professor at the Air Force Institute of Technology. He is a member of the Omega Rho and Alpha Pi Mu honor societies as well as a member of the American Society for Quality Control. Captain Lanning is married to the former Anita Lynn Haines. They have no children, but do have a dog and some houseplants.

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ABSTRACT

This study considers multiple stream systems where it is possible to monitor only a fraction of the total streams at a given time. This situation is of interest in those processes where the speed of production is great and includes a large number of streams, but the ability to monitor the process is not fully automated and unable to keep up with the speed of production.

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Adaptive approaches to system monitoring are applied to multiple stream processes in general and the fractional sampling problem specifically. This represents the first application of adaptive techniques to multiple stream processes. The average time to signal for an adaptively-monitored, fractionally-sampled multiple stream process is developed using a Markov chain procedure. The average time-to-signal results are used to identify promising adaptive sampling schemes for monitoring multiple stream processes using fractional samples. The adaptive fraction approach is shown to give superior results to the fixed fraction scheme and often yields satisfactory results compared with those obtained by sampling all the streams involved in a process.

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DEDICATION

This research is dedicated to those individuals I hold most dear. To my parents, who set my feet on a firm foundation and instilled within me an appreciation of education and an unquenchable hunger for knowledge. Mom and Dad, the apple truly does not fall very far from the tree. To my brother, who brightened these years with mirth and along the way became my best friend. Matthew, I had the time of my life. To my loving wife, without her endless support and encouragement I never would have begun, let alone completed, this effort. Anita, you are my sweetest laughter. To my Lord and Savior, who continuously overlooks my shortcomings and gives me unspeakable joy. Jesus, without you I would be truly lost. Borrowing a line from *Titanic*, "You have saved me in every way a person can be saved."

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My education would be empty without my fellow students. You all have explained and re-explained things which I would never have grasped without you. You have formed the society of our department that makes the program a splendid adventure. I treasure our stories, laughter, anguish, and memories.

A special thanks to the Arizona State University faculty and staff not on my committee. Dr. Dwayne Rollier who introduced me to the department, Dr. Jeffery Cochran who encouraged me to maintain a broader focus on operations research issues, Dr. Matt Carlyle who helped with the dissertation defense, and Ms. Maggie Langely and Ms. Barbie Orcutt who made sure my program of study stayed on track.

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CHAPTER 1

INTRODUCTION

Preliminaries

In the beginning, the use of statistics to monitor a process was presented by Walter A. Shewhart as the *Economic Control of Quality of Manufactured Product* (Shewhart (1931)). Duncan (1986) points out that "Shewhart's ideas set the pattern for application of statistical methods in process control." Shewhart's title also influenced the terms associated with the use of statistical methods, primarily in the parts industry (Box and Luceño (1997)). Terms such as "statistical process *control*" (SPC), "quality *control*," and "*control* charts." Unfortunately, the use of these terms has also led to some confusion. One source of confusion stems from another term within the process industry, "engineering process *control*," or EPC. The confusion stems from the word *control*.

In the realm of SPC, control means to monitor a process and note when corrective action should be taken. This is analogous to the driver of an automobile *controlling* her speed by watching the speedometer. She is actually monitoring her speed and noting when corrective action should take place. For EPC, control means to adjust a process to keep process output variables on target (Janakiram and Keats (1998)). Using the previous example, EPC might be thought of as a vehicle's cruise control. This mechanical device *controls* the automobile by making adjustments to throttle position and engine speed. Table 1-1 shows the differences between SPC and EPC as presented by Messina (1992).

TABLE 1-1. SPC and EPC Comparison (Messina (1992))

	SPC	*EPC*
Philosophy	Minimize variability by detection of and removal of process upsets.	Minimize variability by adjustment of process to counteract process upsets.
Application	Expectation of process stationarity.	Expectation of continuous process drift.
Deployment:		
Level	Strategic	Tactical
Target	Quality Characteristics	Process Parameters
Function	Detecting Disturbances	Monitoring Set Points
Cost	Large	Negligible
Focus	People and Methods	Equipment
Correlation	None	Low to High
Results	Process Improvement	Process Optimization

Source: Messina (1992)

While the definition of *control* might seem a trivial matter, Box and Luceño (1997) tell of hostility between practitioners of SPC and EPC as the division between the two approaches has blurred. In order to alleviate some of tension and confusion, Box and Luceño suggest referring to EPC techniques as *process adjustment*, while using *process monitoring* for typical SPC approaches such as Shewhart charts, exponentially weighted moving average (EWMA) charts, and cumulative sum (CUSUM) charts. Toward that end, this study will use the term *process monitoring* in place of *process control* when referring to SPC (or should that be SPM?) techniques. This is not to say that the word *control* will be taboo in either EPC or SPC situations. Having defined the meaning of *control* in this chapter, this study will use common terms such as *control chart*, *control limit*, and *SPC*.

Research Motivation

Consider a hypothetical organization, the Acme Bottling Company, which started business filling bottles with various beverages many years ago. Being a new business, Acme started out small with only one bottling line consisting of a single fill valve. The management of this fledgling company soon saw the need to statistically monitor their small process to ensure the prevention of both under-filling and over-filling of bottles. They settled on a simple Shewhart $\overline{X}$ chart as a reasonable means for monitoring their process. While the economic viability of this company may be questioned, the methods of constructing and maintaining the requested $\overline{X}$ chart are well understood.

An interesting thing happened as the Acme Bottling Company began to grow. The addition of a second line to accommodate the filling of cans as well as bottles, did not pose a serious problem, but did require an additional $\overline{X}$ chart. Likewise, adding an additional valve to the bottling line was handled satisfactorily by monitoring each valve with separate $\overline{X}$ charts. However, as growth continued and more valves were added to each line, the number of charts required to monitor the process became overwhelming. Since modern technology now allows over 100 valves per filling machine, an alternative approach to monitoring the filling process at the Acme Bottling Company must be considered.

When the Acme Bottling Company started using more than one valve on a given bottling line, their operation became a multiple stream process (MSP). Runger, Alt, and Montgomery (1996) define the MSP as, "A manufacturing process with observed data at

a point in time consisting of measurements from several identical process streams." Similarly, Stephenson (1995) refers to the MSP as, "A process consisting of several identical sub-processes called streams." The modern operational situation of the Acme Bottling Company constitutes a multiple stream process with a large number of streams. While a few streams can be effectively monitored using separate $\overline{X}$ charts for each stream, processes with large numbers of streams require a different technique.

Many processes can be classified as multiple stream processes. Runger, Alt, and Montgomery (1996) identify several examples: thickness measurements taken across a sheet, or web; diameter measurements taken at different heights or radii; measurements of identical features of a single part; measurements from several identical production tools; measurements from identical test instruments; measurements from different locations on a wafer or disk; and measurements from different leads on a printed circuit board. Ott and Snee (1973) discuss MSP monitoring in a situation similar to the example presented earlier in this chapter – filling operations. Figure 1-1 shows an example of two different types of filling operations. The rotary-type filling process is of special interest as it allows very large numbers of streams (often greater than 100 fill valves) operating at very high speeds (up to 1500 filled items per minute).

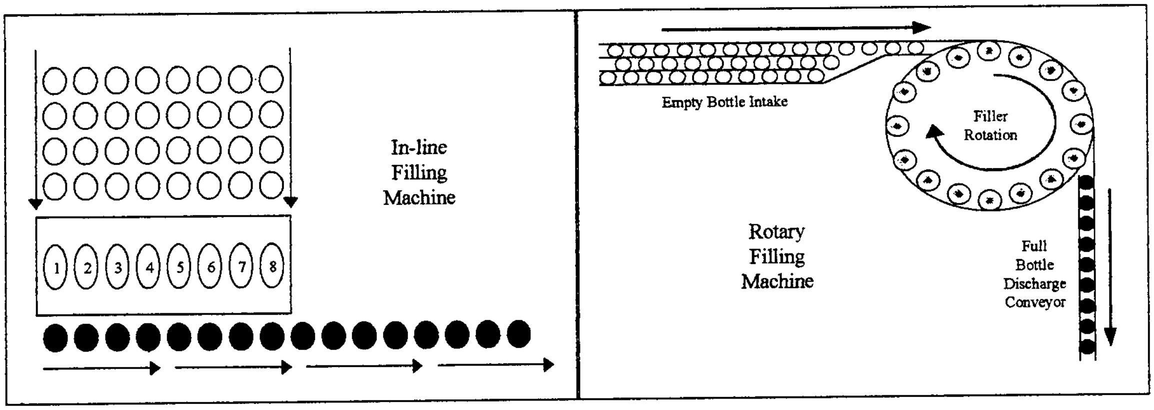


FIGURE 1-1. Sample Filling Operations: (a) In-line Filling, and (b) Rotary Filling

While some methods have been developed to address MSP situations, to date all of these approaches require that samples be comprised of data from each stream in the process. That is, if a filling machine has 8 valves, a sample size of $n = 1$ would include 8 bottles – one from each valve, or stream. In processes with very large numbers of streams, especially those operating at high speeds, it is not always possible to collect samples at a given point in time that include items from each stream. This situation of taking fractional samples in a MSP has not been addressed in the literature and is the focus of this study.

Problem Statement

This study focuses on how to monitor multiple stream processes with a large number of streams where it may not be possible to measure all streams at a given time. To help define the scope of this problem, the issues associated with having a large number of streams will be discussed first and then the situation where only a fraction of the streams can be measured at a given point in time will be addressed.

Before proceeding, it will be advantageous to define some terminology and assumptions of a standard multiple stream process. Assume samples are taken at time t from a process with p streams where n samples from each stream (a sub-sample) are measured. Each measurement then can be thought of as

X_{ijk} where $i = 1, 2, \ldots\ t$ time

$j = 1, 2, \ldots\ p$ streams

$k = 1, 2, \ldots\ n$ sub-samples

Figure 1-2 shows an example from a filling operation with $p = 8$ streams, where each sample taken at a given time has $n = 2$ sub-samples.

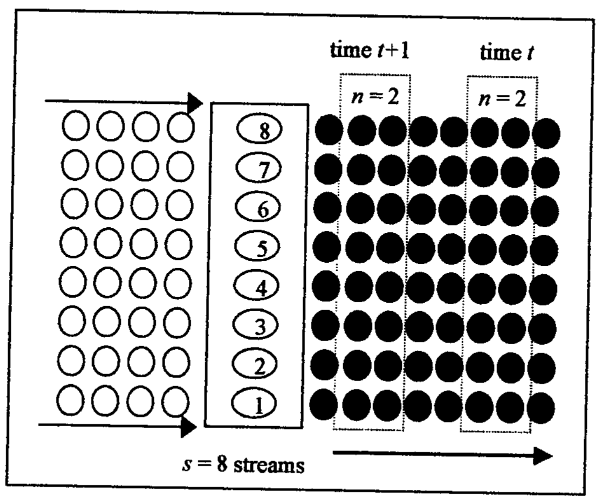


FIGURE 1-2. In-Line Filling Operation with $p = 8$, and $n = 2$

The number of sub-samples will often be, $n = 1$. This is especially true when all items produced are being monitored. To simplify notation, this situation will be assumed, although the following discussion is equally appropriate to averages taken across sub-samples (that is each sample consisting of more than one item from each stream).

There are several problems associated with multiple stream processes involving a large number of streams. The first series of difficulties involves attempts to extend standard process monitoring chart techniques for one or two process streams to processes with large numbers of streams. Monitoring each stream individually would require p control charts to be maintained with p samples plotted at every time t. As the number of streams being monitored increases to more than just a few, this procedure clearly becomes unmanageable.

Besides merely being cumbersome, there are other problems associated with a large number of individual control charts. One problem is that of increased false alarms. For example, the average number of samples taken before a false alarm is signaled for a standard, two-sided $\overline{X}$ chart with an on-target process, is 370. If p streams are being monitored, this false alarm rate increases to one in every $370/p$ samples (Stephenson (1995)). This means more false alarms. So if a high-speed, rotary-type filling machine has 100 fill valves and is monitored using separate $\overline{X}$ charts, the process would generate a false alarm about every fourth rotation of the machine. This is not acceptable. We could compensate by adjusting the location of the chart limits for each individual stream to obtain a desired false alarm rate for the machine as a whole, but we would sacrifice the ability to quickly detect off-target conditions of any single stream.

Another issue associated with the use of individual charts for each stream concerns how the individual charts react to various assignable causes. If an assignable cause affects just one stream, the individual chart will detect this within the limits of its chart parameters. An assignable cause that has a large impact on the mean of all the

streams will generate signals on most of the individual stream charts, but small shifts will be detected singly. For example, given a process with 30 streams, a large shift may cause 20 or more of the individual charts to signal at the same time. On the other hand, a small shift may cause chart 10 to signal on the 10^{th} observation and then charts 6 and 14 to signal on observation 12, and so on. This behavior is indistinguishable from the situation where individual streams are being affected rather than all the streams. It would take a substantial amount of time before the conclusion that all streams had shifted from the target mean could be reached.

Mortell and Runger (1995) raise a third concern regarding the use of individual charts. They point out that if the product variability ($\sigma^2_{\text{product}}$) is large compared with the variability between streams (σ^2_{stream}), the ability of separate charts for each stream to detect a shift of any one stream is nearly impossible. This problem is due to the fact that the control limits for each chart must account for variability of the product as a whole, in addition to the variability of the individual stream. For example the control limits might be

$$\mu \pm 3\sqrt{\left(\sigma^2_{product} + \sigma^2_{stream}/n\right)} \tag{1-1}$$

where μ is the mean of the process . It is clear that if $\sigma_{product}$ is large compared to σ_{stream}, it will be very difficult to catch a shift that impacts only a single stream (Mortell and Runger, (1995)).

This discussion shows that while the MSP with only a small number of streams might be monitored using separate charts for each stream, MSP situations involving large, and very large numbers of streams will require an alternate approach. A chart that monitors the multiple stream process as a whole and signals when an assignable cause impacts the process is desired. The signal should occur whether all the streams are impacted, just one stream, or some subset of streams. The group control chart attempts to provide a solution to this situation.

A group control chart plots only the maximum and minimum values seen across all streams, and identifies which stream generated each maximum or minimum, at every time t. While this significantly reduces the number of charts being monitored, it does not significantly reduce the number of false alarms generated. In fact, if the control limits for the group chart are the same as those used on charts for each individual stream, the false alarm rate will be identical. Obviously, if any single chart had a value large enough to cause a signal on the individual charts, then a chart of maximums with the same limits would also signal.

By noting which streams generated the maximum and minimum values across all streams, a technique of monitoring the runs of these values can be used to identify individual streams that may be off-target. For example, if the same stream on the filling machine is repeatedly producing the maximum fill, there is evidence that this stream may be off-target. A drawback of the runs rule monitoring technique is discussed by Mortell and Runger (1995). They point out that the runs scheme fails to account for the situation where more than one stream shifts from target. If two streams should shift, it is likely

that the maximum (minimum) value will alternate between them. This limitation will reduce the ability of the group control chart to identify situations where more than one stream is off-target. Group control charts and other methods will be discussed in the detail in the next chapter.

The next complication for multiple stream processes involving large numbers of streams is evident in situations where only a subset, or fraction, of the total number of streams can be sampled. While group control charts and other methods have improved the ability to monitor multiple stream processes, none have yet addressed how to best monitor the MSP where only a fraction of the streams can be measured at a given time. An example of such a situation can be found in the now familiar bottling process. A common high-speed filling machine can have over 100 fill valves and is capable of filling thousands of cans, or bottles a minute. At these speeds, sampling all the streams at a given time must be accomplished by machinery which, in some cases, does not yet exist. Instead, samples of a fraction of the streams are taken periodically.

The problem then is to determine an effective method for monitoring multiple stream processes when only a fraction of a large number of streams can be sampled at any given point in time.

Research Goals

The primary goal of this study is to generate a solution to the research problem identified in the previous section. In obtaining the solution, several interim goals will be pursued. Before developing any alternative monitoring techniques, a review of solutions

to similar problems must be made. A necessary first objective, therefore, is to present a comprehensive review of statistical process monitoring techniques with a special focus on their growth and application in situations containing multiple stream processes.

The monitoring of any process by statistical methods requires knowledge of how off-target situations will manifest themselves, and the probabilities associated with their occurrence. These probabilities are often used to develop performance measures that indicate the effectiveness of various methods (often charting techniques) in monitoring specific processes. In order to evaluate the effectiveness of any proposed approach to monitor fractionally sampled multiple stream processes, an appropriate measure of correctly detecting off-target situations will need to be established. The same will be true for identifying associated false alarm rates. These measures will be developed using the likelihood, or probability, of detection.

Whenever any sample is drawn from a population, care must be taken to ensure accurate information about the population can be gleaned from the sample. This will be doubly true for in this case. Not only are we intending to sample from a production population, we will also be sampling from the population of streams. The sampling plan used to build the charts and statistics for monitoring the process will be paramount to any method's success. Developing appropriate fractional MSP sampling plans will be a necessary interim goal.

While this approach may lead to an effective theoretical method for monitoring multiple stream processes using only a fractional sample, the value will be limited unless the method can be applied in practical situations. A final goal will be to present a

representative MSP with a large number of streams, and demonstrate how the process might be monitored using only a fraction of the available streams.

Importance of the Study

The ability to monitor processes involving large numbers of streams continues to grow in importance. As the numbers of streams increase, the challenges associated with monitoring statistical quality increase as well. In some process industries, the machinery that enables greater numbers of streams has advanced faster than the ability to monitor all the streams. Further complicating the situation, the speed of operation also continues to increase. A methodology allowing fractional sampling of a multiple stream process is necessary in these situations.

In addition to sheer size, processes with very large numbers of streams are more likely to have some streams which are correlated and which may arise from different underlying distributions. Likewise, as technology allows more frequent measurement of each data stream, the possibility the data being autocorrelated becomes more likely. Mortell and Runger were able to improve on the group control chart approach by monitoring the range across all streams at time t. While their approach allows for correlation among the streams, their technique, like other control charting schemes, is subject to distortion by autocorrelated data.

The issue of autocorrelated data is an important one and can be a major headache for many statistical charting schemes. These charts rely on an important assumption of independent observations. When this assumption is violated, problems occur. The

principle problem centers on increased false alarm rates. While positively autocorrelated data does help a chart signal more quickly when an assignable cause has impacted the process, it also substantially increases the false alarm rate for on-target processes.

Conditions resulting in autocorrelated data are fairly common. Many processes are driven by inertial forces relating to the physical aspects of the process, flow rates, tank pressure, etc. Advancements in technology allowing inspection of every discrete part manufactured also introduce autocorrelated data in cases where the "sampling interval is short compared with the time constant of the process." (Faltin, et al.(1997)). That is when relatively few items are produced between samples.

While existing methods have been developed for dealing with autocorrelated data in a single stream environment, it is unclear how this would translate to a multiple stream environment. Until methods are developed enabling the direct monitoring of autocorrelated multiple stream processes, a technique using fractional sampling might circumvent some of the problems associated with autocorrelated multiple streams.

Stream correlation implies underlying relationships among the streams. These relationships can arise in many different fashions. When each stream draws from a common resource, be it a common raw product being packaged, or a common pressure supply, variations in the common resource have related impacts among the streams of a process. Some of these relationships may exist across all streams, say a common raw material, and some may affect only a subset of the streams, say a manifold supplying pressure to 10 out of 40 streams. Other relationships may relate to the date of manufacture or last maintenance action, human interaction with the equipment (different

operators, or different shifts), or even relative position within the set of streams. Often these relationships may help the statistical monitoring process. Groups of deviations will make it more likely to catch an assignable cause and will facilitate correction of the problem. For example, if 12 sequential valves on a filling machine fail together, the likelihood of each valve having an internal problem is less than an outside cause.

Clearly there are several issues surrounding multiple stream processes. These MSP issues can usually be split into two broad categories; those processes involving a relatively few, or moderate number of streams, and processes with a large number of streams. If very few streams are involved, each stream can be monitored separately. Most MSP research has addressed the situation involving a moderate numbers of streams where there are too many streams to monitor individually. This research builds on the work presented in the literature for moderate numbers of streams, average run length determination, adaptive monitoring methods, and associated techniques for determining adaptive chart performance to produce original contributions for how to monitor MSPs with large numbers of streams.

Contributions center around processes where only a fraction of the total streams can be monitored, and that are typically influenced by assignable causes that impact all or most of the process streams. This is the first presentation of issues surrounding fractionally sampled multiple stream processes. Specific contributions include the development of a model for determining detection probabilities in fractionally sampled multiple stream processes. This probability model is used to derive associated ARL performance measures. The integration of adaptive sampling schemes to large MSP

problems is also introduced. While adaptive approaches themselves are not new, this work is the first to use adaptive schemes to monitor MSP problems. The final contribution is the construction of a Markov chain method that incorporates the new probability model to measure the performance of adaptive schemes of monitoring fractionally sampled MSPs.

Organization

This study is made up of four primary parts excluding this introduction and the concluding chapter. The Chapter 2 is devoted to the foundation of statistical process monitoring (also called statistical process control, or SPC), and many of the techniques from which multiple stream process monitoring has grown. While some background material will be reserved for later chapters, the bulk of the literature review for this study will be found in Chapter 2.

Chapter 3 develops and discusses the probability of detection associated with fractional sampling from multiple streams. These probabilities are presented in tabular and graphical form as well as the more familiar associated measure of performance, the average run length (ARL). The computer program code used to determine general and specific probability values is included as an appendix to Chapter 3.

Sampling plans are presented in Chapter 4. The emphasis is on adaptive approaches to sampling and how they can be applied to MSP situations. Background information on adaptive processes is presented here as well as examples for each method considered.

Chapter 5 draws the ideas from the previous chapters together in an interesting case study. While the names have been changed, and the numbers altered – the situation presented is genuine. Much of the discussion is hypothetical as the actual process is not prepared to move to the stage of monitoring proposed by this study, at least not yet. Other process improvements need to be made first, but the progression is toward being able to implement a similar scheme to that presented in Chapter 5.

Finally, Chapter 6 concludes the study with a summary of findings and presents several avenues for further research in the area of multiple stream processes in general, and multiple stream processes with very large numbers of streams in particular.

CHAPTER 2

LITERATURE REVIEW

Introduction

Monitoring the quality of manufactured product (often called quality control, or statistical control) is an issue which engineers have been wrestling with for some time. W. A. Shewhart defines the problem as the determination of how much variance should be left to chance (Shewhart, 1931). The ability to anticipate reasonable levels of chance variation, and thereby also recognize unreasonable levels, is fundamental to effective statistical process monitoring. Indeed, Douglas Allan states the purpose of statistical quality monitoring is to assess the amount of chance variability likely to occur and thereby allow the detection of assignable causes of variation (Allan, 1959).

The following section discusses the Shewhart $\overline{X}$ chart to introduce notation and terminology. We will also review the concept of average run length as a measure of the performance of $\overline{X}$ and other charts where the time between subgroups is a constant. The section on Shewhart charts will conclude with a look at several $\overline{X}$ chart enhancements proposed to improve the performance of the Shewhart type charts.

Then we will review some of the charts suggested as alternatives to the Shewhart $\overline{X}$ chart. We will investigate the how these methods work and compare them with one another and with the standard Shewhart $\overline{X}$ chart. Techniques used in special situations will also be examined. Particular attention will be given to multi-variate techniques as

these provide a springboard for discussion of multiple stream processes. This chapter will conclude with a look at how multiple stream processes have been monitored to date.

Shewhart Charts

A common method of monitoring processes and detecting assignable causes of variation is by way of a control chart. The first charts were developed by Walter Shewhart while working for Bell Telephone Laboratories in 1926 (Shewhart, 1931) and are commonly called Shewhart or $\overline{X}$ (X bar) charts. Control charts are a graphical method of displaying process variation on a time scale.

Assuming a process is only being subjected to chance causes of variation, statistical limits can be established within which observations should fall with some desired probability. In addition to control limits and plotted points, the typical Shewhart chart has a center-line (CL) representing the target of the process usually centered between the upper control limit (UCL) and lower control limit (LCL). This type of chart is shown in Figure 2-1.

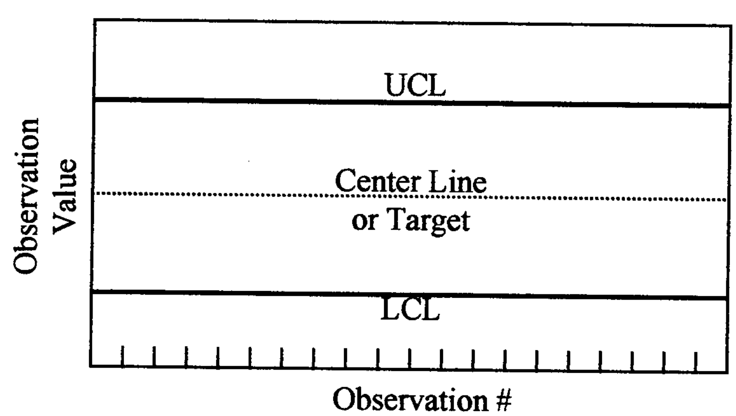


FIGURE 2-1. Typical $\overline{X}$ Chart

Suppose a process to be monitored has a mean, μ_X and variation, σ^2_X. Then the typical Shewhart chart would establish limits and target values of

$$UCL = \overline{\overline{X}} + 3s_X$$
$$CL = \overline{\overline{X}}$$
$$LCL = \overline{\overline{X}} - 3s_X$$

where $\overline{\overline{X}}$ is the grand average and a best estimator of the process mean, μ, and s^2 is the estimated variance. Observed values of the process are plotted on this chart and the chart is monitored for evidence of an assignable cause impacting the variability of the process. Figure 2-2 shows a typical $\overline{X}$ chart with observed values. Values can be recorded singly (in which case $\overline{x} = x$), or in subgroups of a desired size. These are referred to as charts for individuals, and charts for averages, respectively.

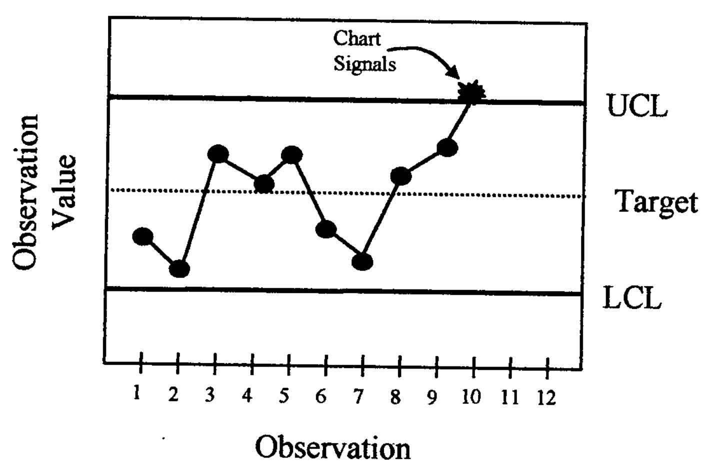


FIGURE 2-2. $\overline{X}$ Chart for Monitoring a Process

Should an event occur which results in a shift of the mean of the process, the points will soon plot beyond the statistical limits causing the chart to signal. This signal indicates an assignable cause may now be influencing process variability and causing the process to be off-target. The 10th data point in Figure 2-2 plots beyond the UCL and therefore gives sufficient evidence that the process may be off-target to warrant some corrective action. Figure 2-3 shows how an upward shift in the process mean of size δ causes the Shewhart chart to signal. As the process mean has increased, there is a greater probability that a sample from the shifted process will plot beyond the UCL.

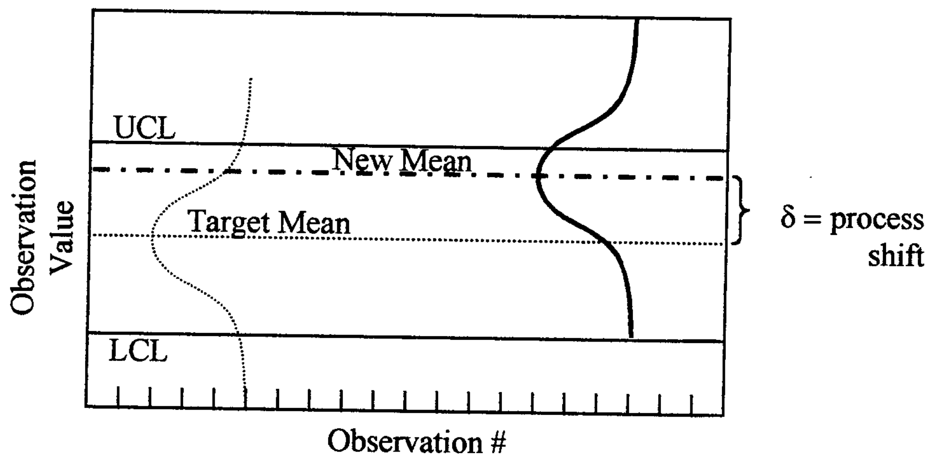


FIGURE 2-3. Distribution of $\overline{X}$ Before and After a Process Shift

Performance Measures. The number of samples required for a chart to signal, after a shift in the process occurs, is called a run length. An average run length (ARL) is simply the expected number of samples taken before the chart signals. The ARL can be computed for various size shifts in the process variability. The ARL

associated with a shift of size zero, or no change in process variability, indicates the likelihood of a false alarm and is called ARL(0).

The determination of the ARL is closely linked to hypothesis testing. Essentially each plotted sample represents a hypothesis test. The null hypothesis, H_0, states the realized sample was drawn from a population matching the target characteristics. When a sample falls within the control limits, we fail to reject H_0 and assume the process is on-target. Should a sample plot beyond the chart limits, we reject H_0 and assume the sample came from a population that does not match the target characteristics. Since we know the sample came from the monitored process, we assume that process has changed, and the change is substantial enough to warrant an investigation into the cause of the process change.

Where we decide to set the chart limits is a critical decision that determines the probability of a chart signaling when the process remains on-target (false alarms) and the risk of failing to detect off-target conditions. These situations are known as type I and type II errors respectively. Table 2-1 shows four possible outcomes of every sample plotted on the control chart and how type I and type II errors occur. Looking back at Figure 2-3 we see that widening the chart limits decreases the chance of a type I error. Unfortunately, this action will also increase the probability of a type II error. If we make the chart limits more narrow, the opposite holds true (that is we increase the likelihood of type I errors while reducing type II errors).

TABLE 2-1. Type I vs. Type II Errors

	Sample within chart limits	*Sample beyond chart limits*
Process On-target	Correct ID Continue monitoring	False Alarm Type I error
Process Off-target	Fail to Detect Problem Type II error	Correct ID Chart Signals

In most circumstances, type II errors are considered more troublesome than type I errors. The rationale for this is seen in Table 2-1. The type II errors indicate situations where we assume the process is operating normally, when in fact there is a problem. This results in wasted product or lost revenue until the situation finally generates a signal. (Note: this need not necessarily be a signal from a control chart. If the process was allowed to run off-target long enough, the signal may be from numerous irate customers!) While false alarms (type I errors) are unwelcome and may take some time to straighten out, at least unacceptable production is not taking place.

The ability of a control chart to minimize type II error while preventing the type I error from inflating beyond a reasonable level is often measured using average run lengths. A Shewhart chart with limits at ±3 standard deviations of process variation has an ARL(0) of about 370. This means that, on average, a Shewhart chart will signal once in every 370 trials for an on-target process. For Shewhart charts where the plotted points are independent, the ARL can be found easily using

$$ARL = \frac{1}{p} \tag{2-1}$$

where p is the probability that a point will exceed the chart limits. Note that p is simply $1 - q$ where q is the probability that $\overline{X}$ lies between the lower and upper chart limits.

$$q = P\{LCL \leq \overline{X} \leq UCL\} \tag{2-2}$$

For a Shewhart chart with upper and lower limits at 3σ, q is simply

$$\begin{aligned} q &= P\{Z \leq 3\} - P\{-3 \leq Z\} \\ &= 0.99865 - (1 - 0.99865) \\ &= 0.9973 \end{aligned}$$

and since $p = 1 - q$ we have $p = 1 - 0.9973$ which yields $p = 0.0027$. Applying Equation 2-1 and taking the inverse of p we obtain the stated result

$$ARL(0) = \frac{1}{p} = \frac{1}{0.0027} \cong 370$$

The ARL for off-target situations depends on both the size of the process shift and the size of the sub-sample averaged for each data point. Average run lengths for Shewhart charts with various sub-samples at several different shift sizes are shown in Table 2-2.

TABLE 2-2. 2-sided Shewhart Chart Average Run Lengths with Various Sample Sizes

Shift = δ	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 20$
0.0	370.38	370.38	370.38	370.38	370.38	370.38	370.38
0.5	155.22	90.65	60.69	43.89	33.40	12.83	4.50
1.0	43.89	17.73	9.76	6.30	4.50	1.77	1.08
1.5	14.97	5.27	2.91	2.00	1.57	1.04	1.00
2.0	6.30	2.32	1.47	1.19	1.08	1.00	1.00
2.5	3.24	1.42	1.10	1.02	1.00	1.00	1.00
3.0	2.00	1.12	1.01	1.00	1.00	1.00	1.00

The performance of alternative chart techniques is usually determined by comparison with the ARL values of the Shewhart chart. When other monitoring techniques are discussed later in this study, comparisons will be made in this fashion. Runger and Pignatiello (1991) sum up the idea nicely.

> When the waiting time (time between samples) is constant, the performance of various control charts can be compared by considering their respective ARLs. If two charts have the same ARL when the process is operating in an on-target state, then the two charts can be compared by examining their ARLs for various off-target states. If one chart yields smaller ARLs for all off-target states, then it is clearly better on an ARL basis.

Runger and Pignatiello point out that the time between samples needs to be constant for this type of comparison. This implies that performance measures other than the ARL are occasionally warranted. Reynolds, Amin, Arnold, and Nachlas (1988) point out that, with a fixed time interval between samples, the ARL can be converted to an expected, or average time to signal by simply multiplying the ARL by the fixed time interval. The average time to signal (ATS) can also be computed for monitoring schemes that allow the time interval between samples to vary. The ATS performance measure can

then be used to compare those monitoring techniques with the standard Shewhart technique.

A potential drawback of the ATS is its assumption that the assignable cause responsible for shifting the process off-target occurs at time 0. In practice the process may start out on-target, and shift at some time $t \neq 0$. Therefore, Reynolds, Amin, Arnold, and Nachlas (1988) suggest the process should be measured using the adjusted average time to signal (AATS) when the interval between samples is variable. Costa (1997) also uses this approach and defines the AATS as the average time from the shift in the process mean until a signal is generated. Runger and Pignatiello (1991) and Runger and Montgomery (1993) also use a similar approach, but prefer the term steady-state ATS to AATS.

To determine the AATS, Costa makes use of the average time of the cycle (ATC) defined as the average amount of time from start of production until the first signal following a shift in the process. Assuming an exponential distribution with parameter λ for the occurrence of an assignable cause, he obtains AATS = ATC – $1/\lambda$.

Costa goes on to define two more terms for purposes of comparing chart performances. He says control charts should be compared which require, on average, an equal number of samples and an equal number of items inspected via those samples. Charts are then compared using the average time of the cycle (ATC), the average number of samples (ANS), and the average number of items (ANI).

One potential problem with the ATS is that it does not differentiate between sampling plans that call for short time intervals between samples and those with long intervals. For example, if scheme 1 samples the process every 10 minutes while scheme 2 samples every 10 hours, scheme 1 will have a much shorter ATS for process shifts since scheme 2 cannot have an ATS of less than 10 hours. Reynolds, Amin, Arnold, and Nachlas (1988) prefer to define yet another measure for schemes allowing variable intervals between samples to account for this. They define the average number of samples to signal (ANSS) as the expected value of the number of samples taken from the start of the process until the time the chart signals.

Rather than use the ANSS performance measure, or defining still another new measure, Runger and Pignatiello (1991) opt to "calibrate" their charts. This is accomplished by setting the adaptive chart's average time between samples when the process is on-target equal to the fixed time between samples of the Shewhart chart. Runger and Pignatiello use a common average time between samples of one hour. The standard Shewhart chart is then considered to always gather samples at 1-hour intervals thereby allowing direct comparisons of alternative monitoring techniques.

For monitoring approaches where the sample size is not constant, the ANSS is not sufficient as it does not account for variable sample sizes. Costa (1994), as well as Park and Reynolds (1994) address this issue by defining yet more performance measures. Tagaras (1998) sums up these measures using the following definition:

> Average Number of Observations to Signal (ANOS): the expected value of the number of inspected items from the start of the process (or the occurrence of the assignable cause) to the time when the chart signals.

While many performance measures have been used to evaluate chart performance and effectiveness, the average run length measure remains most important. Any new approach to monitoring a given process will always be compared against standard statistical monitoring techniques. As the standard approaches are measured in terms of ARL, a conversion or comparison between a new technique and its associated performance measure to a standard format will be required.

Sensitizing Rules. Several enhancements to the Shewhart chart have been proposed over the years in an attempt to improve chart performance. Most of the enhancements suggested involve having the chart signal based on various sensitizing, or runs rules. Some of the most popular rules were introduced by the Western Electric Company in 1958 and so these rules are often known as the Western Electric run rules. Duncan (1986) defines a run as a succession of items of the same class. As an example, consider the performance of the stock market. A succession of market closings below the previous day's close would be considered a run of down days. A runs rule can be derived by considering the probability of interesting and rare data histories.

As it pertains to statistical process monitoring, a runs rule takes into account the likelihood of a particular plotted point by comparing it to the points immediately preceding it. That is, rather than considering only the most recent sample point, we also take into consideration the recent run of data points and note any "unnatural" data patterns. The Western Electric Company's *Statistical Quality Control Handbook* (1958) describes characteristics of unnatural data patterns.

Unnatural data patterns always involve the absence of one or more of the characteristics of a natural pattern. For example:

(1) Absence of points near the centerline produces an unnatural pattern known as "Mixture."

(2) Absence of points near the control limits produces an unnatural pattern known as "Stratification."

(3) Presence of points outside of the control limits produces an unnatural pattern known as "Instability."

Montgomery (1996) sums up the motivation for runs rules by pointing out that, in addition to looking for data points which plot beyond chart limits, we are also interested in situations where the data exhibit non-random behavior. For example, if the data are behaving in a truly random fashion we would expect roughly half the data points to lie above the center line, and half below. If, however, we realize an unusually high percentage, say 90 percent, of the data points above the center-line we would conclude the data pattern appears very non-random.

In addition to the familiar control limits, runs rules schemes use other limits often called warning limits, or thresholds. To monitor the run history on a standard Shewhart $\overline{X}$ chart, warning limits are usually added at $\pm 2\sigma$ and $\pm 1\sigma$. These limits divide the chart into 3 zones above the center-line and 3 zones below the center line. Runs rules are then established making use of these zone definitions. Figure 2-4 shows the standard Shewhart chart is modified in this fashion.

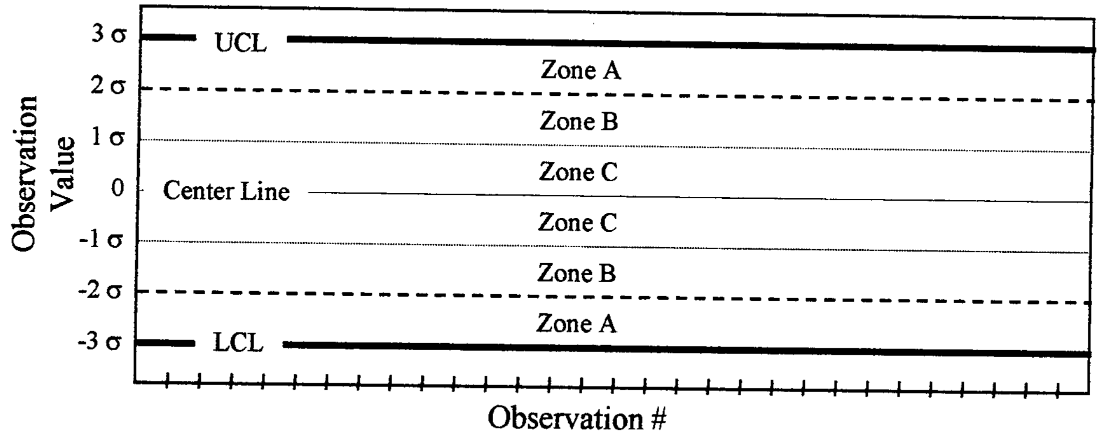


FIGURE 2-4. Shewhart Chart with Additional Warning Limits for Runs Rules

The Western Electric Handbook (1958) provides several decision rules for potential nonrandom chart patterns. Some of these rules are listed in Table 2-3.

TABLE 2-3. Typical Runs Rules

Rule #	*Chart Signals if ...*
1	... a single data point plots outside Zone A, the 3σ chart limits.
2	... two out of three consecutive data points plot beyond Zone B, the 2σ warning limits.
3	... four out of five consecutive data points plot at or beyond Zone C, 1σ from center.
4	... eight consecutive data points plot on one side of the center line.
5	... any obvious nonrandom pattern is seen within the data points.

The first rule is recognized as the standard method by which the Shewhart chart signals. Rules 2, 3, and 4 are illustrated in Figures 2-5, 2-6, and 2-7 respectively.

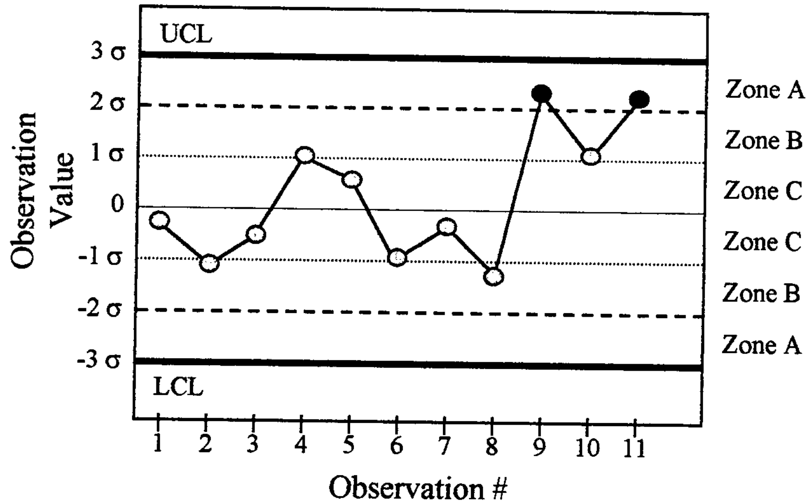


FIGURE 2-5. Runs Rule #2: 2 of 3 Sequential Points Plot in Zone A

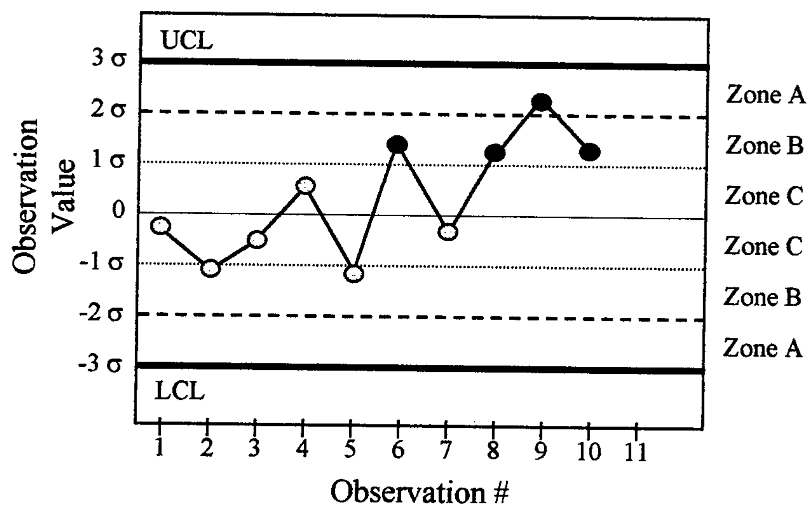


FIGURE 2-6. Runs Rule #3: 4 of 5 Sequential Points Plot in Zone B or Beyond

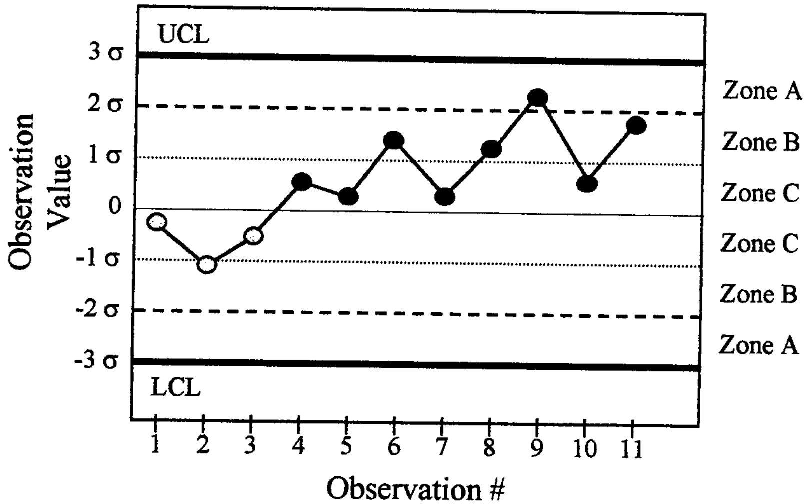


FIGURE 2-7. Runs Rule #4: 8 Sequential Data Points Plot Above the Center Line

Rule 5 is less well defined, but some nonrandom patterns are easily recognized. Figure 2-8 shows a pattern of high, negative correlation, while Figure 2-9 shows positive correlation. Note that in each case the charts would not signal using the other runs rules.

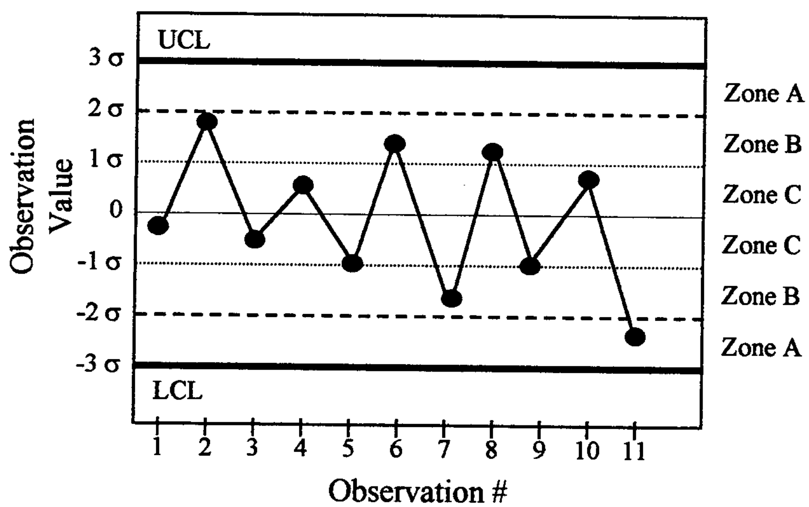


FIGURE 2-8. Runs Rule #5: Data Points with High Negative Correlation

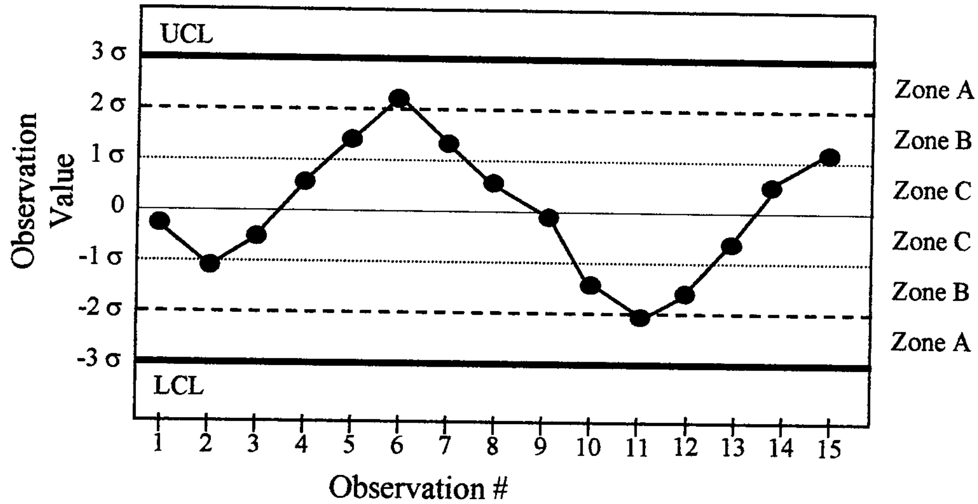


FIGURE 2-9. Data Points with High Positive Correlation

Enhancing the Shewhart chart with run rules makes the chart more sensitive to detecting off-target conditions. Unfortunately, as more rules are added to the chart, the likelihood of false alarms also increases. In fact, if a control chart is constructed using r rules and each rule i is independent and has an associated probability of generating a false alarm, α_i, then the overall false-alarm probability, α, is given by

$$\alpha = 1 - \prod_{i=1}^{r} (1 - \alpha_i) \tag{2-3}$$

Montgomery (1996) points out that the assumption of independence between each rule is probably not accurate, and so Equation 2-3 should be considered an approximation of the overall false-alarm probability.

The exact performance results for Shewhart charts augmented with various runs rules are enumerated by Champ and Woodall (1987). Table 2-4 is a condensed version of

Champ and Woodall's Table 1 and shows how the runs rules improve the performance of the Shewhart chart especially for small shifts of the process mean. Note that the values for a shift of 0 have also decreased indicating higher false-alarm rates.

TABLE 2-4. ARLs for Shewhart Charts Enhanced with Various Runs Rules

Shift δ	*Rule 1*	*Rules 1, 2*	*Rules 1, 3*	*Rules 1, 4*	*Rules 1, 2, 3*	*Rules 1, 2, 4*	*Rules 1, 3, 4*	*Rules 1, 2, 3, 4*
0.0	370.40	225.44	166.05	152.73	132.89	122.05	105.78	91.75
0.6	119.67	57.92	33.99	33.64	28.70	27.49	23.15	20.90
1.0	43.89	20.01	12.66	14.58	10.95	11.73	10.19	9.22
1.6	12.38	6.21	5.24	7.03	4.54	5.27	5.01	4.41
2.0	6.30	3.65	3.68	4.89	3.14	3.50	3.65	3.13
2.6	2.90	2.13	2.43	2.81	2.07	2.13	2.43	2.07
3.0	2.00	1.68	1.89	1.99	1.67	1.68	1.89	1.67

Source: Champ and Woodall (1987)

Range Charts. When monitoring a process variable it is standard practice to monitor both the process mean and variation (Montgomery (1996)). The Shewhart $\overline{X}$ chart is used to monitor the process mean, but can often be improved by the addition of a chart designed to monitor process variability.

To illustrate the need for both charts, consider the following example. In the manufacture of automobile tires, the thickness of the tire tread is a key factor in determining tire quality. Monitoring the thickness of tires during production is important as tires that are too thin will wear out before the warranty expires and tires that are thicker than necessary mean wasted material and lower production yields. The target production distribution might look like that shown in Figure 2-10.

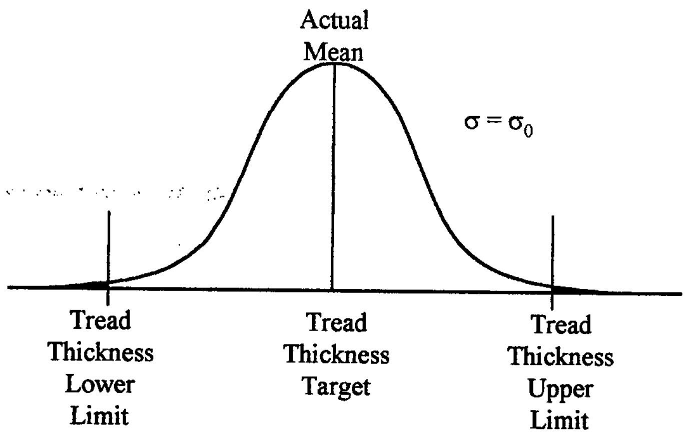


FIGURE 2-10. Target Distribution for Tire Production

The Shewhart $\overline{X}$ chart used to monitor the mean of the production process should quickly identify situations where the process mean shifts off-target as shown in Figure 2-11.

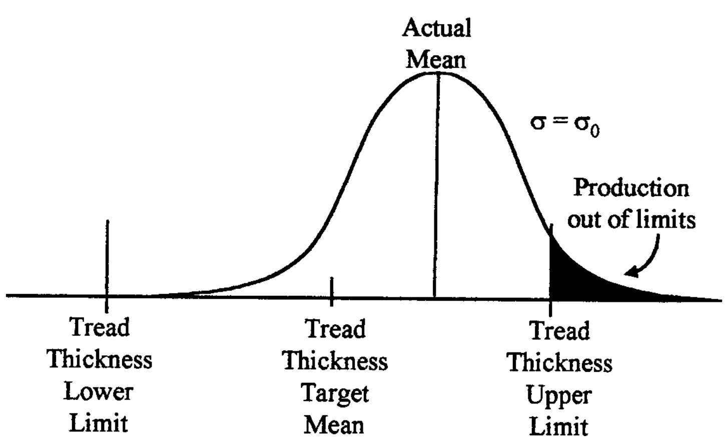


FIGURE 2-11. Upward Shift in the Production Mean = More Out-of-limits Tires

The $\overline{X}$ chart fails to perform adequately when the mean remains on-target, but the process variation increases. In such cases the sample averages will tend to be drawn

toward the target value, but a greater percentage of the process will be operating in the tails of the distribution. This situation is illustrated in Figure 2-12.

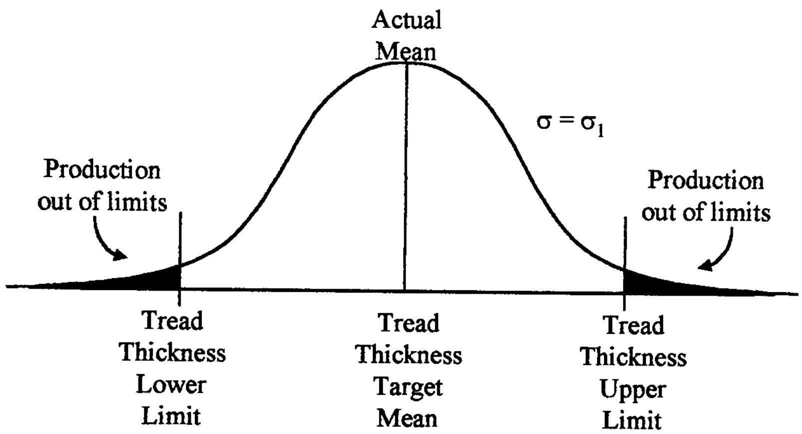


FIGURE 2-12. Tire Production Mean On-target, but with Increased Process Variation

The situation shown in Figure 2-12 can be effectively monitored using a chart for the process range, called an R-chart. A sample range is defined as the difference between the largest observation, x_{max}, and the smallest observation, x_{min}. That is,

$$R = x_{\max} - x_{\min} \tag{2-4}$$

Since the range (R) contains information about the spread of the sample data, R can be used to estimate the process standard deviation. If we define $\overline{R}$ as the average range taken over several samples, then the standard deviation can be estimated by

$$\hat{\sigma} = \frac{\overline{R}}{d_2} \tag{2-5}$$

where d_2 is the mean of the relative range (Montgomery (1996)). The relative range is the random variable $W = R/\sigma$ whose distribution depends on the number of observations contained in a sample. Values for d_2 can be found in quality control texts such as Montgomery (1996), and Duncan (1986) among others. Some of the values for d_2 are given in Table 2-5 along with the relative efficiencies Montgomery reports for using $\overline{R}$ to estimate σ rather than s^2.

TABLE 2-5. Values of d_2 and the Relative Efficiency of $\overline{R}$ vs. s^2 for Various n

n	d_2	*Relative Efficiency*
2	1.128	1.000
3	1.693	0.992
4	2.059	0.975
5	2.326	0.955
6	2.534	0.930
10	3.078	0.850

Now that we know how to compute the range, we can monitor the range along with monitoring the process mean. To monitor the range we establish R as the center line, and define range limits using an estimate of the standard deviation of R, σ_R. Montgomery shows that σ_R can be estimated by

$$\hat{\sigma}_R = d_3 \frac{\overline{R}}{d_2} \tag{2-6}$$

where d_3 is the standard deviation of the relative range $W = R/\sigma$ and is a function of the sample size. Like d_2, d_3 can be found in most quality control texts. Now the upper range limit (URL) and lower range limit (LRL) using 3σ control limits are

$$URL = \overline{R} + 3\hat{\sigma}_R$$
$$LRL = \overline{R} - 3\hat{\sigma}_R \qquad (2\text{-}7)$$

With these parameters an $\overline{R}$ chart can be constructed to monitor the process variability. It is worth noting that for many processes the lower range limit, LRL, will be negative in which case the chart should be used with simply an upper range limit. Furthermore, evidence of a process exceeding the LRL will often not result in production problems, but may represent an opportunity to reduce overall process variability.

Alternative Monitoring Techniques

In addition to the Shewhart $\overline{X}$ charts, other methods of statistically monitoring processes have been developed. Notably the cumulative sum (CUSUM) chart first suggested by Page (1954) and the exponentially weighted moving average (EWMA) introduced by Roberts (1959). In many, if not most, applications these charting schemes are superior to the Shewhart method.

Cumulative Sum (CUSUM). A major drawback of the Shewhart chart is the fact that it only incorporates data from the current time period. The runs rules try to fill this gap, but other methods have been developed which often work better. Montgomery

(1996) tells how the CUSUM incorporates information from the sequence of data points by plotting the cumulative sums of the deviations of the sample values from the specified target. The basic idea of the CUSUM is to note the difference between the most recent sample mean, $\bar{x}_j$ and the intended process mean, or target value, μ_0. Then, as the name implies, this difference is added to the sum of previous differences as in

$$S_i = \sum_{j=1}^{n} (\bar{x}_j - \mu_0) \tag{2-8}$$

where i is the observation number. Rather than actually plotting this value, however, most CUSUM techniques make use of a threshold, or reference value, K, which must be overcome before the CUSUM increased. Since the CUSUM approach was initially developed to monitor processes for a shift in only one direction (Montgomery (1996)), the following quantities are defined to monitor upper (S_H) and lower (S_L) one-sided process shifts respectively.

$$S_H(i) = \max[0, \bar{x}_i - (\mu_0 + K) + S_H(i-1)] \tag{2-9}$$

$$S_L(i) = \min[0, \bar{x}_i + (\mu_0 + K) - S_L(i-1)] \tag{2-10}$$

where K is the reference value, typically chosen midway between the target mean value and the value of the mean we want to be able to detect. The reference value serves to keep the value of the CUSUM equal to zero unless the current sample mean is substantially greater than (less than) the target mean. The two-sided CUSUM chart is

derived through the combining of two, one-sided procedures using upper and lower reference values, K_H and K_L respectively.

The primary advantage of the CUSUM is realized in situations where the shift in the process mean is small, or when the sample size (n) of each data point is 1, also known as *charts for individuals*. A second advantage is that the process shift is often easy to detect by simple inspection of the plotted data. Another advantage is that CUSUM method also allows for fast initial response (FIR). This technique allows the chart to have a head-start toward signaling whenever the chart is initially started, or restarted following an assignable cause. This option is desirable in that when a process restarts, there is some chance that the problem which triggered the signal may not have been corrected, or that the corrective action further affected the process.

Brook and Evans (1972) used Markov chains to find ARLs for the CUSUM method. Champ and Woodall (1987) also used Markov chains to thoroughly compare run lengths of CUSUM charts against Shewhart charts augmented with runs rules. The Markov chain approach to determining average run length will be described in some detail later in this document. Goel and Wu (1971) give a nomogram on the design of CUSUM charts with specified average run lengths.

CUSUM design methods are reviewed by Gibra (1975), Goel (1981), and Woodall (1986) among others. Many other authors have studied various aspects of the CUSUM chart. Lucas (1985) describes design and implementation procedures for a counted data CUSUM, also called CUSUM for attributes. Gan (1991a and 1994)) looked

at optimal CUSUM control chart schemes. Finally, Hawkins (1993b) discusses a technique for making the CUSUM robust through Winsorization.

Exponentially Weighted Moving Average (EWMA). The exponentially weighted moving average monitoring technique also outperforms the Shewhart chart when small shift sizes are to be detected. The EWMA technique introduced by Roberts (1959) has been studied by many authors; including Lucas and Saccucci (1990), Crowder (1989), Ng and Case (1989), and Gan (1991b). Crowder identifies two situations for applying the EWMA technique. The first is a white noise process occasionally affected by shifts in the process mean where the EWMA is used to monitor the process. The second situation is characterized by gradual drifts in the process mean and affords the EWMA an opportunity to forecast process behavior. Box, Jenkins, and Reinsel (1994) as well as Hunter (1986) provide detailed discussions of the latter situation.

Montgomery (1996) points out that the EWMA is roughly equivalent to the CUSUM in performance, although it may be simpler to operate. The EWMA is defined as

$$z_t = \lambda \bar{x}_t + (1-\lambda) z_{t-1} \tag{2-11}$$

where $0 < \lambda \leq 1$ is a constant and $z_0 = \bar{\bar{x}}$. The control limits for the EWMA are

$$UCL = \bar{\bar{x}} + 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)n}} \tag{2-12}$$

$$LCL = \bar{\bar{x}} - 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)n}} \tag{2-13}$$

Other work using the EWMA approach includes a study using the EWMA to monitor process standard deviations by Crowder and Hamilton (1992). MacGregor and Harris (1993) introduce the notion of an exponentially weighted moving variance (EWMV) chart and an exponentially weighted mean squared deviation (EWMS) to monitor process variation.

Multivariate Charts. To this point we have assumed a need to monitor only one process variable. This is often not the case. Many situations exist in which two or more characteristics of the same item need to be monitored. Montgomery (1996) gives an example of a bearing with both an inner and outer diameter. When confronted with multiple variables that need to be monitored simultaneously, several univariate control charts could be used. This approach can, however, give poor results in certain situations.

The first problem is one of increased false alarm rates. If we use 3 separate charts to monitor 3 variables, where each chart has an ARL(0) of 370, the false alarm rate of the 3 charts combined will be approximately 123. Clearly it does not take very many variables before the false alarm rate will be unacceptably high. Montgomery (1996) shows that, in general, for p statistically independent quality characteristics monitored using $\overline{X}$ charts with type I probability of α, the true probability of type I error for the joint procedure is

$$\alpha' = 1 - (1 - \alpha)^p \qquad (2\text{-}14)$$

The probability that all the means will plot inside their independent chart limits simultaneously (that is the probability of not getting a false-alarm) is

$$\mathrm{P}\{\text{all } p \text{ means within chart limits}\} = (1-\alpha)^p$$

Another problem arises if the variables have a high degree of correlation. In this situation, the use of several univariate charts can give poor performance (Ryan, (1989)). Ryan (1989) discusses a multivariate control chart scheme based on the T^2 distribution work done by Hotelling (1947). In the single variable case

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \tag{2-15}$$

where t follows a t distribution. Now letting $\mu = \mu_0$ and squaring the distribution

$$t^2 = \frac{(\bar{x}-\mu_0)^2}{s^2/n}$$

$$= n(\bar{x}-\mu_0)(s^2)^{-1}(\bar{x}-\mu_0)$$

This result can be generalized allowing k variables as

$$T^2 = n(\bar{\mathbf{x}}-\boldsymbol{\mu}_0)S^{-1}(\bar{\mathbf{x}}-\boldsymbol{\mu}_0) \tag{2-15}$$

where

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_k \end{bmatrix} \quad \boldsymbol{\mu}_0 = \begin{bmatrix} \mu_1^0 \\ \mu_2^0 \\ \vdots \\ \mu_k^0 \end{bmatrix}$$

now when $\mu = \mu_0$, T^2 follows an F distribution so values of T^2 can be plotted on a control chart with an appropriate F-value as a control limit. Ryan suggests comparing T^2 with

$$\frac{p(n-1)}{n-p} F_{\alpha(p,n-p)} \tag{2-16}$$

where α is chosen so that $\alpha/2p = 0.00135$ which is the 3σ value for a univariate chart.

Figure 2-13 shows how two univariate $\overline{X}$ control charts for a hypothetical data set might miss an out of control situation. A multivariate chart using T^2 plotted against an appropriate F limit is shown in Figure 2-14 for the same data.

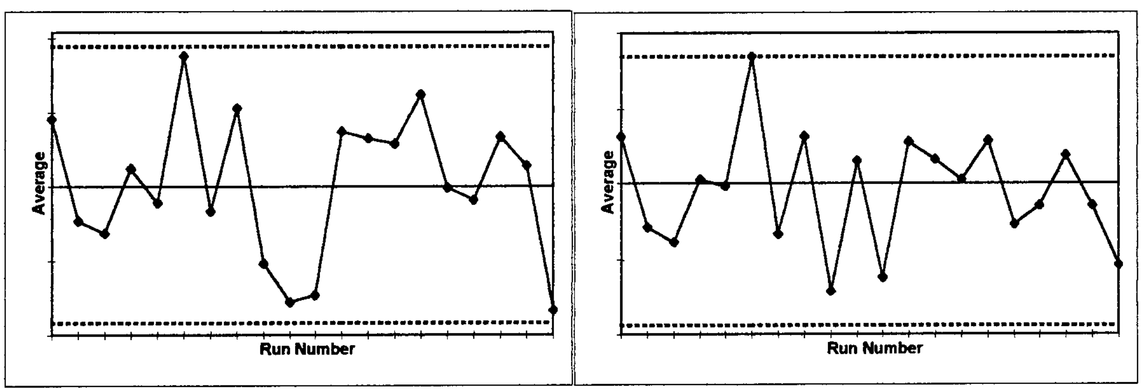


FIGURE 2-13. $\overline{X}$ Charts for Two Hypothetical Variables

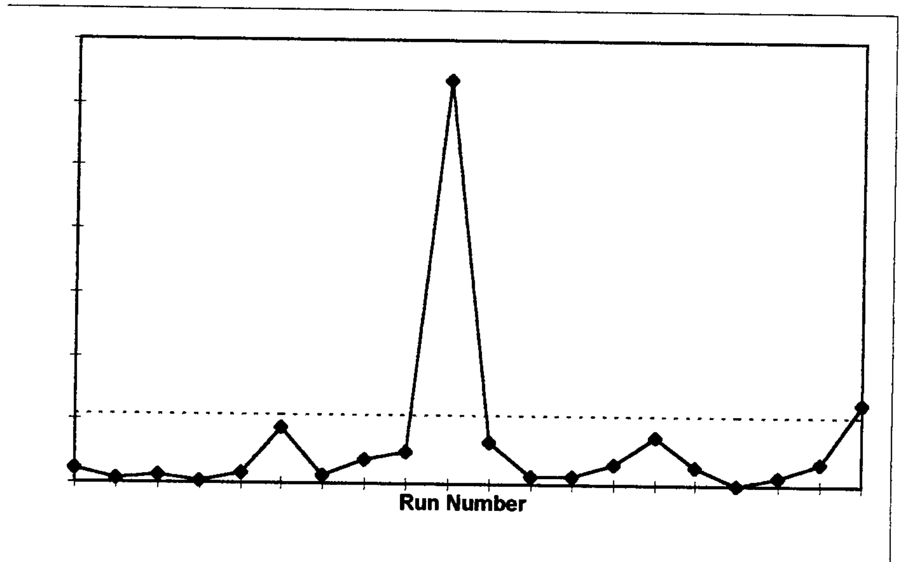


FIGURE 2-14. Multivariate Control Chart (T^2 vs. *time*)

Since most quality control applications involve the monitoring of more than one variable, the multivariate problem continues to be an important issue. Interesting contributions to the multivariate problem have been made by several authors. Tracy, Young, and Mason (1992) looked specifically at the multivariate problem for individuals. A CUSUM approach to multivariate processes was developed by Pignatiello and Runger (1990). Lowry, Woodall, Champ, and Rigdon (1992) extended the process to the EWMA chart. Rigdon (1995b) used integrals to develop ARL(0) run lengths for multivariate EWMA charts. The identification of off-target characteristics in the multivariate arena was investigated Doganaksoy, Faltin, and Tucker (1991). Hawkins (1991) used regression-adjusted variables to monitor multivariate processes. An attempt to help interpret signals from multivariate charts by decomposing the T^2 distribution was made by Mason, Tracy, and Young (1995). Finally, a new approach using projections and the U^2 multivariate chart was proposed by Runger (1996).

Multiple Stream Processes

A special monitoring problem arises when there are several identical sub-processes, or streams. This situation is referred to as a *multiple stream process* (MSP). The statistics obtained when sampling from an on-target MSP are generally independent and identically distributed. An example of a multiple stream process might be a machine used to fill several bottles at one time (Ott and Snee, 1973). Each group of bottles enters the machine where they are filled simultaneously and then move on down the line as in Figure 2-15.

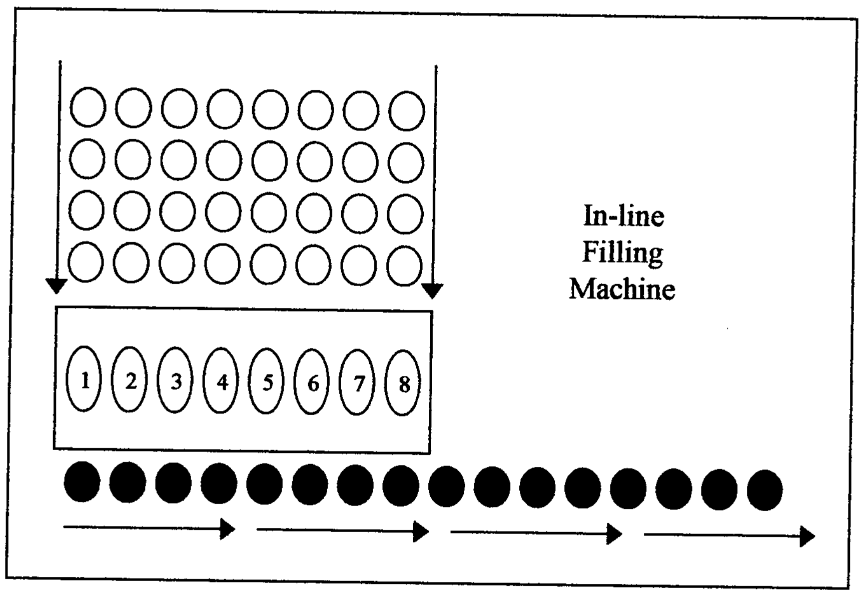


FIGURE 2-15. In-line Filling Machine

A control chart could be monitored for each stream, but as mentioned in the previous section, the false alarm rate increases dramatically as the number of streams becomes large. An alternative approach to this problem, developed over 50 years ago (Nelson (1986)), is called group control charts.

Group Control Charts. To construct a group control chart, only the largest and smallest values observed need to be plotted. If the maximum and minimum values are within acceptable limits, all streams will be in control since all the other observations must lie between the maximum and minimum values. Figure 2-16 shows how four separate charts would be combined as one group chart.

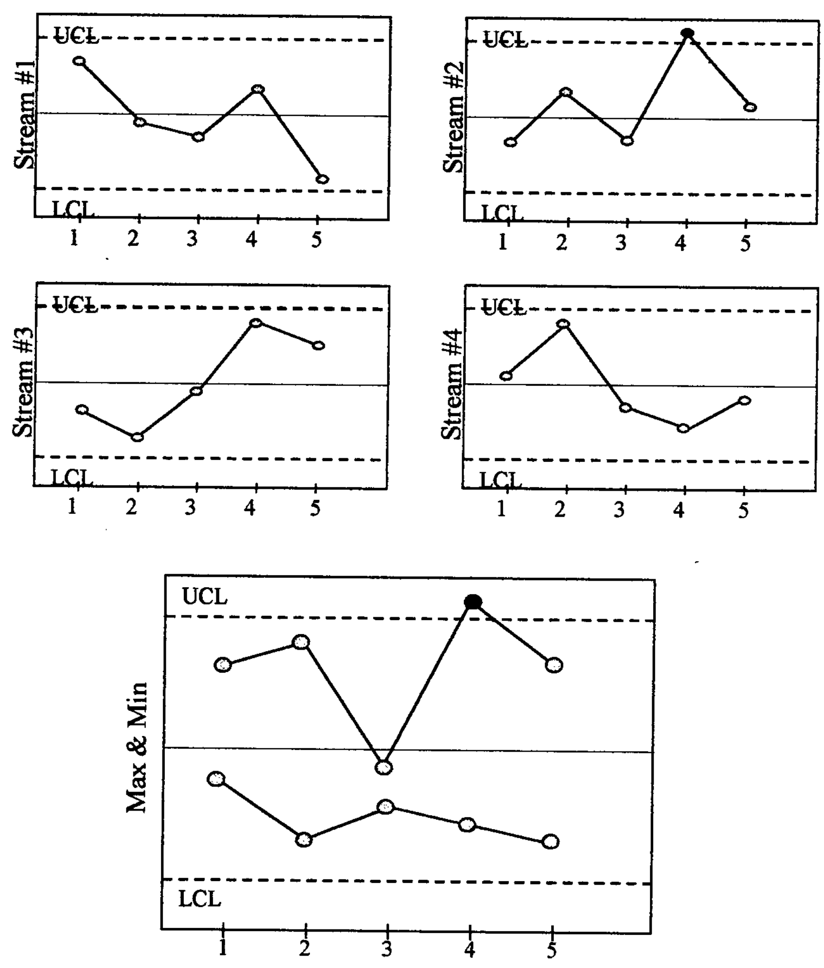


FIGURE 2-16. Separate $\overline{X}$ Charts vs. a Group Control Chart

Two possible assignable cause scenarios immediately present themselves. The first involves an assignable cause which shifts the mean of all the streams (Nelson, 1984). The second is a shift that affects only one stream. The first case is handled directly by the control chart. If a shift affects all the streams, then the maximum and minimum will likewise be impacted and the chart will signal in the usual fashion. For the second case a special runs rule is used. The stream that generates the maximum (or minimum) value should change in a random fashion. If the same stream generates an unusual number of maximums in a row, then that particular stream is likely to be out of control (Nelson, 1986).

The use of the runs rule for a multiple stream process raises a practical difficulty. Mortell and Runger (1995) point out that since the runs scheme is a discrete process, the difference in ARL(0) for 3 maximums in a row can be substantially different than that for 4 maximum values in a row. Nelson (1986) and Montgomery (1996) give the following equation for determining the ARL of the runs scheme for an on-target process

$$ARL = \frac{p^r - 1}{p - 1} \tag{2-17}$$

where p is the number of streams and r is the run length of a particular stream as the maximum (minimum) value. For example, if a process has 15 streams, a runs rule based on 3 maximum (minimum) in a row yields an ARL(0) of 121.8. This may result in unacceptably high numbers of false alarm indications. Moving to a scheme based on 4

maximum (minimum) in a row results in an ARL(0) of over 1800. Unfortunately this will result in long detection times for small to moderate shift sizes.

Another runs rule limitation pointed out by Mortell and Runger is where more than one stream (say two) shifts, but not all the streams. In this case it is reasonable that the two streams would alternate among themselves the maximum value and not cause the runs rule to signal.

One of the problems associated with maintaining a separate chart for each stream is the resultant high false alarm rates. This problem is not solved by use of the group control chart. Since the group control chart is plotting maximums and minimums and using the same control limits as the separate charts, if any one of the individual charts would have signaled, the maximum chart will necessarily also signal. One possible means of improving this situation is to set the chart limits further apart. This makes sense for a chart of maximums (minimums) as the last (first) order statistic is expected to be well beyond the standard $\pm 3\sigma$ limits. In fact, for a sample of size 10, the value likely to be exceeded with probability 0.0027 is 3.64.

Other MSP Methods. Stephenson (1995) uses a method of least favorable conditions to develop conservative ARLs for the group control chart approach to multiple stream processes. The ARLs are validated using material presented by Woodall and Reynolds (1983) on the application of Markov chains to the sequential probability ratio test. Stephenson also attempts to address one of the runs scheme problems by proposing an *n-1 out of n in a row* rule. This approach allows some flexibility back into the model,

but also adds complexity since the runs rule is no longer a simple counting procedure. Furthermore, the original runs problem still exists for multiple stream processes with extremely large numbers of streams. The concept of least favorable conditions will continue to apply as a method for generating conservative ARL estimates.

The multiple stream process problem is nicely laid out in Mortell and Runger (1995). In addition to defining the current state of the problem, they offer a new solution by suggesting control charts based on the range of observed data. Mortell and Runger specifically address the important role of how the variance is allocated in the model. The relative size of these variance components plays a key role in deciding how to approach the MSP problem. In their approach to the MSP problem, Mortell and Runger use a two pronged attack. They use a classic $\overline{X}$ chart for the average across all streams in the process to detect a shift affecting all streams. In order to detect shifts affecting only one, or just a few streams, they monitor the range of the process at each sample. That is rather than plot the maximum (x_{max}) and minimum (x_{min}) values as in a group chart, they plot $R = x_{max} - x_{min}$. Clearly if all streams shift at once, the range will not be affected, but the $\overline{X}$ chart should quickly signal. On the other hand, if only one stream shifts, the $\overline{X}$ chart may not detect the shift, but the range chart should. In an advantage over the group chart, if two or more streams shift, the range chart is more likely to signal, rather than less likely as in the group chart's runs scheme.

Finally Mortell and Runger show how the application of other multivariate techniques can be applied to the MSP problem. This suggests the possibility of entirely new approaches to the problem in areas like principal components, cluster analysis, and

factor analysis. In addition to these new approaches, Mortell and Runger also indicate that their approach will work in an adaptive environment like that proposed by Prahbu, Montgomery, and Runger (1994).

Runger and Alt (1996) discuss the application of principal component analysis of the multivariate problem by customizing multivariate control charts. Building on this effort and Mortell and Runger's work, Runger, Alt, and Montgomery (1996) suggest approaching the MSP problem using principal components. They use the first two principal components to control the two kinds of partitioned variation in the multiple stream process. Jackson (1980) gives an excellent tutorial on the use of principal components.

Summary

This literature review provides a history of the foundations of statistical process monitoring and multiple stream process issues and serves to identify the large amount of work still possible in this area. One of the prime areas as yet undeveloped is the very large number of streams problem. This problem involves both correlated and autocorrelated data, systematic sub-sampling and adaptive techniques, and the impact of non-identical streams. Whether there are a large number of streams or not, the issue of non-identical streams is an area as yet unsolved.

The focus of the following investigation is to address those situations where a large number of streams are contained in the process and not all the streams can be sampled at a given time. The study will commence with a look at how to determine the

probability of detecting an off-target condition when only a fraction of the streams are sampled. The application of adaptive sampling methods will also be pursued and how to implement adaptive techniques in a fractionally sampled process.

CHAPTER 3

PROCESS MONITORING USING FRACTIONAL SAMPLES

Introduction

In traditional statistical process monitoring (SPM), the performance of various monitoring methods is compared using the average run length (ARL) of each proposed scheme. As discussed in the previous chapter, false alarm rates, (also called ARL(0)), are usually equated for each competing scheme and then various off-target ARL results are compared, and conclusions made, regarding the performance of each method.

We also saw that adaptive monitoring situations tend to complicate matters of comparison by using performance measures other than the ARL. In these situations, a conversion is necessary to directly compare the non-ARL measured chart with the performance of established charting methods. Runger and Pignatiello (1991) accomplish this by having the average, on-target time between samples equal the fixed sampling interval of Shewhart type charts. We will see how a special case of the multiple stream process (MSP) generates a further complication in ARL definition and chart comparison.

In addition to considering MSPs where all streams shift simultaneously, or cases where exactly one stream shifts, some recent work has addressed MSPs where more than one stream shifts (see Mortell and Runger (1995) and Runger, Alt and Montgomery (1996)). However, no discussion has been given to processes where only a fraction of the total streams are sampled.

In certain MSP situations it may not be feasible to measure all the streams in the process at each sample. Instead only a fraction, or subset of the streams is measured. In

such cases it may be easier to consider a probability of detection for the given number of streams sampled rather than an average run length. Since the ARL is, in effect, a detection probability, once the probability of detection for a fractionally sampled MSP has been established, we should be able to compare results with known ARLs for standard charts.

Background

Average Run Length. Recall from the previous chapter that the average run length refers to the run length of the chart used to monitor a process. Thus the ARL is the average number of points plotted on the chart before an off-target situation is signaled. For Shewhart charts where the plotted sample data points are assumed to be independent, the ARL can be found using the formula given by Montgomery (1996) and others:

$$ARL = \frac{1}{\alpha} \tag{3-1}$$

where α is the probability that a point will fall beyond the chart limits, or the probability of detecting a shift in the process. Note that α is simply $1 - \beta$ where β is the probability that a given sample mean, $\bar{x}$, lies between the lower and upper chart limits. For a Shewhart chart with upper and lower limits at 3σ, $\beta = 0.9973$ and, since $\alpha = 1 - \beta$, we have $\alpha = 1 - 0.9973$ which yields $\alpha = 0.0027$.

Now if we take the inverse of α we obtain the familiar ARL result for the average number of points plotted on the Shewhart $\overline{X}$ chart before a false alarm is indicated, ARL(0).

$$ARL = \frac{1}{\alpha} = \frac{1}{0.0027} \cong 370$$

Of more importance to the information in this chapter, we can also say that the probability of generating a signal in any given sample is 0.0027 – a probability of detection (a false alarm detection in this case).

Adaptive Sampling. The appeal of using a probability of detection rather than the ARL becomes clear when we consider adaptive sampling schemes. When we vary the time between samples, and to some extent, the size of the sample, the ARL doesn't serve very well. Recall that the ARL gives an average number of plotted points on the chart before a signal is anticipated. If the time between those plotted points is allowed to vary, the ARL no longer translates directly to the amount of time expected before signaling.

Several authors have defined new performance measures to use in place of the ARL. Some of these suggestions were spelled out in the previous chapter. In each case, the new performance measure is related to a probability of detection. Assuming the samples obtained are independent, simply taking the inverse of the performance measure will yield a value analogous to a detection probability for each item sampled, or measured, depending on the definition of the associated performance measure. For

example, Costa (1997) uses the adjusted average time to signal (AATS) in place of the ARL. He defines the AATS as "The average time from the process mean shift until the chart produces a signal." If we consider the inverse of the derived AATS, and assume the AATS is reported in hours, we could think of the result as the probability that the process mean shift will produce a signal in any given hour – a probability of detection.

Multiple Stream Processes. Several additional issues are raised when working with a multiple stream process. When sampling from a MSP, each sample is generally assumed to contain equal numbers of measurements from each stream in the process. That is each sample from a 25-valve filling machine is assumed to contain groups of 25 measurements – one or more from each valve. In some processes it is not feasible to take samples across all streams at a given point in time. In such instances a fraction of the streams are often sampled. For example, if the hypothetical 25-valve filling machine is operating at a high rate of speed with samples being collected by hand, fractional samples may be taken in groups of, say, 5 at a time.

A special concern in monitoring multiple stream processes is that while an assignable cause may affect all streams equally, this is not necessarily true. In fact one of the unique aspects of the MSP problem is that individual streams or clusters of streams can move off-target independently of one another. In addition to detecting instances where all streams shift off-target, we also desire to detect and identify individual off-target streams quickly whether they occur singly or in groups.

Answering the question of how to best organize fractional samples from a MSP to achieve rapid detection of off-target conditions all the while maintaining a low false alarm rate is the stated goal of this study. To accomplish this we may want to vary both the fractional sample size and the time between these sub-samples. This points to a need for a performance measure other than an average run length. In addition, taking fractional samples of multiple stream processes begs for an approach using probabilities of detection.

The probability of detection measure needed should allow for fractional sampling schemes involving combinations of all, some, or none of the streams being off target. Once the probabilities associated with various sample schemes have been identified, sampling plans can be developed combining several different fractional sampling schemes to achieve a desired level of protection.

Probability of Detection for Fractional Samples

Determining the probability of detection for a multiple stream process in which the streams are fractionally sampled involves two distinct computations. The first is the probability of obtaining a specific sequence of streams while the second is the probability of signaling given a specific sample sequence. Before we start, let's define some terms and assumptions.

Assume we have a multiple stream process consisting of a number of identical product streams. The measurements from each stream are in equal units and have the same target value, μ_0, and standard deviation, σ. Furthermore we will assume that while

the process is on target, the measurements from each stream are independent, identically distributed normal random variables. We will also assume the process has a large number of streams and the sampling limitations are such that we are not able to sample all the streams at a given time, t. Given this, it is also reasonable to assume that the number of items gathered from each stream sampled is, $n = 1$. For this process we now define the following parameters.

p = number of streams in the process

μ_i = mean of the i^{th} stream $\quad i = 1 \ldots p$

s = number of streams sampled $\quad s \leq p$

g = number of groups of streams with unique k values $\quad j = 0 \ldots g, g \leq p$

k_j = size of the shift associated with the j^{th} group (in units of σ) $\quad k_0 = 0$

m_j = number of streams in the j^{th} group

m_0 = number of on-target streams

m_j^* = number of streams from the j^{th} group contained in s

m_0^* = number of on-target streams in s

To better understand each of these terms, a hypothetical fractional sampling situation is shown in Figure 3-1. This fictional process represents a filling operation using a 16 valve rotary filling machine ($p = 16$). Samples are limited to 5 items at a time due to cart capacity limitations ($s = 5$). Currently 5 valves are overfilling. These 5 valves can be separated into 2 groups ($g = 2$). The first group, containing valves 3 and 6 ($m_1 =$ 2), are overfilling by an average of 1 process standard deviation each ($k_1 = 1.0$). The second group includes 3 valves; valves 2, 9, and 14 ($m_2 = 3$); each overfilling an average

of 2 standard deviations ($k_2 = 2.0$). The remaining valves are all on-target ($m_0 = p - m_1 - m_2 = 11$ valves).

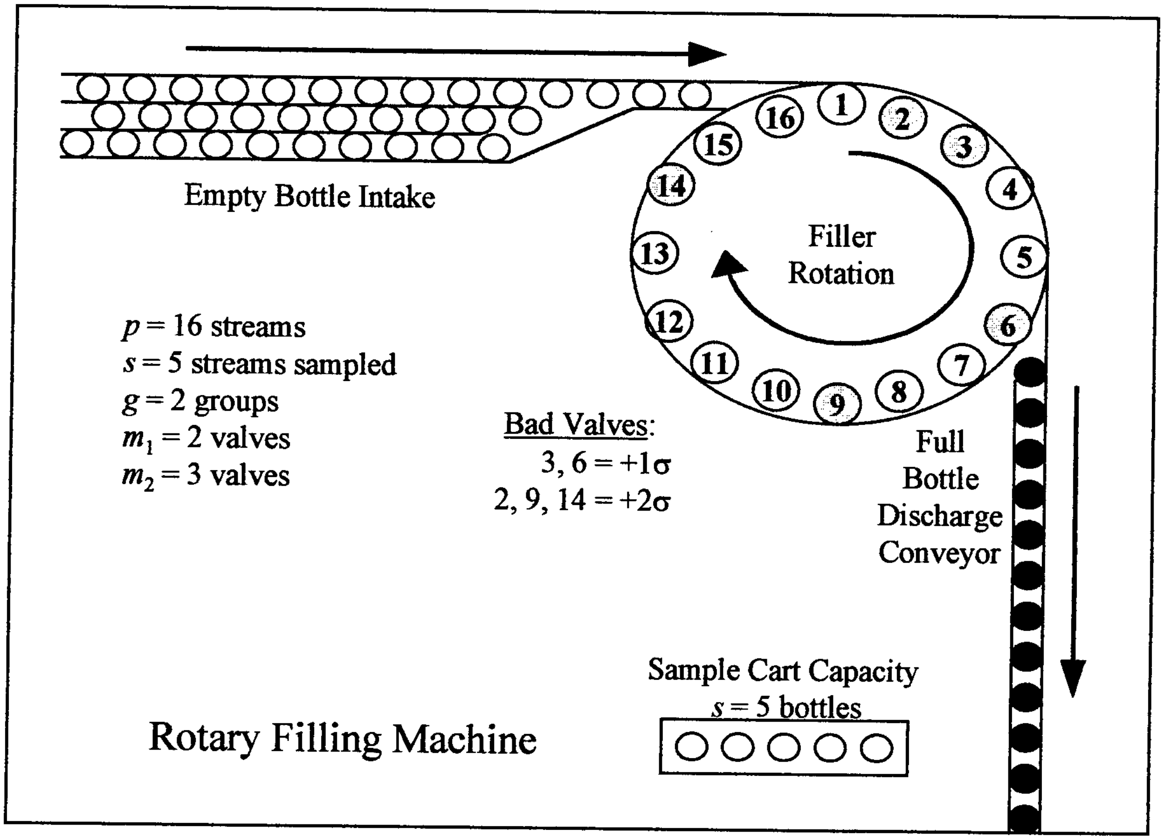


FIGURE 3-1. Sample Filling Operation and Standard Notation

Note that many combinations of good and bad streams are possible using a sample size of 5. We could catch 5 on-target streams, 5 off-target streams, or any combination in between. Assume a random sample is drawn capturing items from valves (streams) 3, 7, 9, 14, and 15. In this case, $m_1^* = 1$ (valve 3), $m_2^* = 2$ (valves 9 and 14), and $m_0^* = 2$ on-target valves (valves 7 and 15).

Sequence Probability. To find the probability of obtaining a specific sequence we need to consider all possible sequences that can be obtained for the fractional sample size, given a total number of streams as well as specific numbers of off-target streams. First we will consider the probability of obtaining the required number of on-target streams. This is accomplished by considering all possible ways of obtaining a sample containing m_0*on-target streams from a total of m_0 on-target streams. This result is then divided by all the possible ways of obtaining a sample of size s from p streams. This, of course, is simply

$$\frac{\binom{m_0}{m_0^*}}{\binom{p}{s}} \tag{3-2}$$

All off-target sub-group possibilities are found in a like manner and when combined with the on-target results we obtain the following equation.

$$P_{sequence} = \frac{\prod_{j=0}^{g}\binom{m_j}{m_j^*}}{\binom{p}{s}} \tag{3-3}$$

This is the probability of obtaining exactly m_1* streams of shift size k_1, m_2* streams of shift size k_2, …, m_Q* streams of shift size k_Q, and m_0* on-target streams in a fractional sample of s streams from a total of p streams.

Now that we have determined the probability of obtaining a particular sequence, we need the probability that this sequence will generate a signal on the chart. This result is developed next.

Signal Probability. The probability of obtaining a signal is the same as 1-β, where β is the probability of not obtaining a signal. This β risk is defined as

$$\beta = P\{LCL \le \bar{x} \le UCL | \mu = \mu_0 + k\sigma\} \tag{3-4}$$

where $\bar{x}$ is distributed $\bar{x} \sim N\left(\mu, \sigma^2 / \sqrt{s}\right)$, μ_0 is the target mean, σ^2 is unknown but constant variance, s is the size of the sample, and k is the shift size.

We can now rewrite β as

$$\beta = \Phi\left[\frac{UCL - (\mu_0 + k\sigma)}{\sigma / \sqrt{s}}\right] - \Phi\left[\frac{LCL - (\mu_0 + k\sigma)}{\sigma / \sqrt{s}}\right] \tag{3-5}$$

and letting $LCL = \mu_0 - 3\frac{\sigma}{\sqrt{s}}$, $UCL = \mu_0 + 3\frac{\sigma}{\sqrt{s}}$ we have

$$\beta = \Phi\left[\frac{\mu_0 + 3\frac{\sigma}{\sqrt{s}} - (\mu_0 + k\sigma)}{\sigma / \sqrt{s}}\right] - \Phi\left[\frac{\mu_0 - 3\frac{\sigma}{\sqrt{s}} - (\mu_0 + k\sigma)}{\sigma / \sqrt{s}}\right] \tag{3-6}$$

We are able to use $\mu+k\sigma$ as the mean of the sample by making use of the linear combination of independent normal variates.

Now using this information and working backwards we obtain

$$\beta = \Phi\left[\frac{\mu_0 + 3\frac{\sigma}{\sqrt{s}} - \frac{1}{s}\sum_{i=1}^{s}\left(\mu_{0_i} + k_i\sigma\right)}{\sigma/\sqrt{s}}\right] - \Phi\left[\frac{\mu_0 - 3\frac{\sigma}{\sqrt{s}} - \frac{1}{s}\sum_{i=1}^{s}\left(\mu_{0_i} + k_i\sigma\right)}{\sigma/\sqrt{s}}\right] \quad (3\text{-}7)$$

Keeping μ_0 constant, but allowing k_i to vary gives

$$\beta = \Phi\left[\frac{\mu_0 + 3\frac{\sigma}{\sqrt{s}} - \left(\mu_0 + \frac{\sigma}{s}\sum_{i=1}^{s}k_i\right)}{\sigma/\sqrt{s}}\right] - \Phi\left[\frac{\mu_0 - 3\frac{\sigma}{\sqrt{s}} - \left(\mu_0 + \frac{\sigma}{s}\sum_{i=1}^{s}k_i\right)}{\sigma/\sqrt{s}}\right]$$

$$= \Phi\left[\frac{\mu_0\sqrt{s}}{\sigma} + 3 - \frac{\mu_0\sqrt{s}}{\sigma} - \frac{\sqrt{s}}{s}\sum_{i=1}^{s}k_i\right] - \Phi\left[\frac{\mu_0\sqrt{s}}{\sigma} - 3 - \frac{\mu_0\sqrt{s}}{\sigma} - \frac{\sqrt{s}}{s}\sum_{i=1}^{s}k_i\right]$$

$$= \Phi\left[3 - \frac{\sqrt{s}}{s}\sum_{i=1}^{s}k_i\right] - \Phi\left[-3 - \frac{\sqrt{s}}{s}\sum_{i=1}^{s}k_i\right] \quad (3\text{-}8)$$

Finally, by letting $\sum_i k_i/s = \bar{k}$ we obtain a β risk for a specific sequence of

$$\beta = \Phi\left[3 - \bar{k}\sqrt{s}\right] - \Phi\left[-3 - \bar{k}\sqrt{s}\right] \quad (3\text{-}9)$$

Since β is the probability of not signaling, the desired P_{signal} is

$$P_{signal} = 1 - \left(\Phi\left[3 - \bar{k}\sqrt{s}\right] - \Phi\left[-3 - \bar{k}\sqrt{s}\right]\right)$$

$$= \Phi\left[-3 - \bar{k}\sqrt{s}\right] - \Phi\left[3 - \bar{k}\sqrt{s}\right] \tag{3-10}$$

This result is useful for determining a probability of detection in a multiple stream environment as we are particularly interested in allowing situations where some, but not all of the streams shift and therefore the sample contains streams with different expected values.

Note that if we allow all s streams in the sample to have the same k equation 3-7 reduces to the more familiar result

$$\beta = \Phi\left(3 - k\sqrt{s}\right) - \Phi\left(-3 - k\sqrt{s}\right) \tag{3-11}$$

Detection Probability. Knowing the sequence probability and the signal probability, we can determine a probability of detection by combining equations 3-3 and 3-10 to obtain

$$P_{detection} = \frac{\prod_{j=0}^{g} \binom{m_j}{m_j^*}}{\binom{p}{s}} \cdot \left(\Phi\left[-3 - \bar{k}\sqrt{s}\right] - \Phi\left[3 - \bar{k}\sqrt{s}\right]\right) \tag{3-12}$$

This result yields a probability of detection for a specific sequence. Given a fractional sample size and number of off-target streams along with their associated means, every possible sequence must be determined and an associated $P_{detection}$ can be

computed for each. Summing over all possible sequences determines the overall probability of detection for each sample.

Let's look at an example using terms as defined in the previous section. Say we have a process containing $p = 16$ streams where 5 of the streams are off-target. Let the off-target streams fall into $g = 2$ groups, $m_1 = 2$ streams with a shift of $k_1 = 1\ \sigma$ above target and $m_2 = 3$ streams with a shift of $k_2 = 2\ \sigma$ above target, leaving $m_0 = 11$. At each sample point assume we take a fractional sample of $s = 5$ streams. This situation results in the 12 possible sample combinations shown in Table 3-1. For each combination a sequence probability and signal probability are determined using equations 3-3 and 3-10 respectively. A total probability for each combination is then found using equation 3-12. Summing over all combinations yields the probability of detection for this situation.

TABLE 3-1. Possible Sample Combinations and Associated Probabilities

#	m_0^*	m_1^*	m_2^*	*Sequence Probability*	*Signal Probability*	*Total Probability*
1	5	0	0	11%	0%	0.0%
2	4	1	0	15%	1%	0.1%
3	4	0	1	23%	2%	0.4%
4	3	2	0	4%	2%	0.1%
5	3	1	1	23%	5%	1.1%
6	3	0	2	11%	11%	1.3%
7	2	2	1	4%	11%	0.4%
8	2	1	2	8%	22%	1.7%
9	2	0	3	1%	38%	0.5%
10	1	2	2	1%	38%	0.3%
11	1	1	3	1%	55%	0.3%
12	0	2	3	0%	72%	0.0%
					Probability of Detection =	6.12%

If we assume this sample plan will continue for all future samples, and that the samples obtained are independent, we can obtain an associated ARL by simply inverting the probability of detection. For the previous example we would obtain

$$ARL = \frac{1}{0.0566} \cong 17.7$$

While this may seem like an arduous procedure, the computer can be used to quickly enumerate all possible fractional sample combinations and apply equation 3-11 to each. A program written in Visual Basic for use inside Microsoft® Excel is described in the Appendix 3B at the end of this chapter.

Performance Measure and Tables & Graphs

The computer program referred to in the previous section was used to generate multiple tables of detection probabilities for various combinations of total streams (p) and number of off-target streams (m). These results are gathered into the tables found in Appendix 3A. Since the probability of detection is not convenient for comparison with known monitoring techniques, tables are also given showing the associated average run lengths. Finally, several average run lengths are graphed showing how different fractional combinations impact process monitoring abilities.

Now that we can determine the probability of detection for any given sample, we should be able to construct an appropriate sampling plan to achieve a desired level of confidence in catching any off-target condition. The tables and graphs in Appendix 3A

give an initial look at how various fractional sampling combinations might be used to effectively monitor a process where all streams cannot be sampled at a given time. The next chapter will investigate alternative sampling plans to determine the best course of action.

Summary

A potential limitation of the method discussed in this chapter is its dependency on independent data streams. For many processes the streams may be autocorrelated (stream 1 at time *t* correlated with stream 1 at time *t+1*), or cross-correlated (stream 1 correlated with stream 2), or both.

The issue of potential correlation is resolved in two ways. First, since we are concerned about a system where we are unable to sample all the streams at a given time, autocorrelated data is not likely to be a concern. Even if samples are taken very frequently, a different subset, or fraction of the total streams is sampled in each time period. This procedure will put enough time between samples from the same streams to remove much of the autocorrelation contained in the process. Second, the data from the streams sampled at time *t* are averaged to provide a single data point. This can be thought of as a form of batching to remove the effects associated with stream-to-stream correlation. Finally, if the time between samples is large compared with the time constant of the process, an assumption of independence of the sample averages is reasonable (Mortell and Runger(1995)).

APPENDIX 3A

ARL TABLES & GRAPHS

Probability of Detection where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of s = 1 of p

# of Off-Target Streams

	p = 5		10		15		20		30		50	
m_1^*	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
1	0.007	0.034	0.005	0.018	0.004	0.013	0.004	0.010	0.003	0.008	0.003	0.006
2	0.011	0.065	0.007	0.034	0.005	0.023	0.005	0.018	0.004	0.013	0.004	0.009
3	0.015	0.096	0.009	0.049	0.007	0.034	0.006	0.026	0.005	0.018	0.004	0.012
4	0.019	0.127	0.011	0.065	0.008	0.044	0.007	0.034	0.005	0.023	0.004	0.015
5	0.023	0.159	0.013	0.081	0.009	0.055	0.008	0.042	0.006	0.029	0.005	0.018
6	--	--	0.015	0.096	0.011	0.065	0.009	0.049	0.007	0.034	0.005	0.021
7	--	--	0.017	0.112	0.012	0.075	0.010	0.057	0.007	0.039	0.006	0.025
8	--	--	0.019	0.127	0.013	0.086	0.011	0.065	0.008	0.044	0.006	0.028
9	--	--	0.021	0.143	0.015	0.096	0.012	0.073	0.009	0.049	0.006	0.031
10	--	--	0.023	0.159	0.016	0.107	0.013	0.081	0.009	0.055	0.007	0.034
15	--	--	--	--	0.023	0.159	0.018	0.120	0.013	0.081	0.009	0.049
20	--	--	--	--	--	--	0.023	0.159	0.016	0.107	0.011	0.065
25	--	--	--	--	--	--	--	--	0.019	0.133	0.013	0.081
30	--	--	--	--	--	--	--	--	0.023	0.159	0.015	0.096
35	--	--	--	--	--	--	--	--	--	--	0.017	0.112
40	--	--	--	--	--	--	--	--	--	--	0.019	0.127
45	--	--	--	--	--	--	--	--	--	--	0.021	0.143
50	--	--	--	--	--	--	--	--	--	--	0.023	0.159

Average Run Length where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of $s = 1$ of p

p

# of Off-Target Streams m_1^*	5		10		15		20		30		50	
	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
1	148.9	29.5	212.4	54.7	247.6	76.4	270.0	95.3	296.8	126.6	322.4	171.8
2	93.2	15.4	148.9	29.5	186.0	42.6	212.4	54.7	247.6	76.4	285.5	111.9
3	67.8	10.4	114.6	20.2	148.9	29.5	175.1	38.3	212.4	54.7	256.1	82.9
4	53.3	7.8	93.2	15.4	124.1	22.6	148.9	29.5	186.0	42.6	232.2	65.9
5	43.9	6.3	78.5	12.4	106.5	18.3	129.5	24.0	165.4	34.9	212.4	54.7
6	--	--	67.8	10.4	93.2	15.4	114.6	20.2	148.9	29.5	195.7	46.7
7	--	--	59.7	8.9	82.8	13.2	102.8	17.5	135.4	25.6	181.4	40.8
8	--	--	53.3	7.8	74.6	11.6	93.2	15.4	124.1	22.6	169.1	36.2
9	--	--	48.1	7.0	67.8	10.4	85.2	13.7	114.6	20.2	158.4	32.5
10	--	--	43.9	6.3	62.2	9.4	78.5	12.4	106.5	18.3	148.9	29.5
15	--	--	--	--	43.9	6.3	56.3	8.4	78.5	12.4	114.6	20.2
20	--	--	--	--	--	--	43.9	6.3	62.2	9.4	93.2	15.4
25	--	--	--	--	--	--	--	--	51.5	7.5	78.5	12.4
30	--	--	--	--	--	--	--	--	43.9	6.3	67.8	10.4
35	--	--	--	--	--	--	--	--	--	--	59.7	8.9
40	--	--	--	--	--	--	--	--	--	--	53.3	7.8
45	--	--	--	--	--	--	--	--	--	--	48.1	7.0
50	--	--	--	--	--	--	--	--	--	--	43.9	6.3

Probability of Detection where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of $s = 0.5\,p$

p

# of Off-Target Streams m_1^*	5		10		15		20		30		50	
	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
1	0.006	0.018	0.004	0.009	0.004	0.007	0.003	0.006	0.003	0.005	0.003	0.004
2	0.018	0.113	0.009	0.041	0.007	0.025	0.006	0.018	0.005	0.012	0.004	0.008
3	0.049	0.376	0.020	0.135	0.013	0.073	0.010	0.049	0.007	0.028	0.005	0.016
4	0.113	0.718	0.041	0.319	0.025	0.175	0.018	0.113	0.012	0.062	0.008	0.031
5	0.222	0.930	0.078	0.564	0.044	0.338	0.030	0.222	0.018	0.120	0.011	0.056
6	--	--	0.135	0.787	0.073	0.539	0.049	0.376	0.028	0.209	0.016	0.096
7	--	--	0.216	0.923	0.117	0.731	0.076	0.552	0.043	0.329	0.022	0.154
8	--	--	0.319	0.980	0.175	0.871	0.113	0.718	0.062	0.469	0.031	0.230
9	--	--	0.439	0.996	0.249	0.950	0.162	0.847	0.087	0.613	0.042	0.325
10	--	--	0.564	1.000	0.338	0.985	0.222	0.930	0.120	0.743	0.056	0.432
15	--	--	--	--	0.809	1.000	0.638	1.000	0.397	0.993	0.190	0.893
20	--	--	--	--	--	--	0.930	1.000	0.743	1.000	0.432	0.996
25	--	--	--	--	--	--	--	--	0.941	1.000	0.704	1.000
30	--	--	--	--	--	--	--	--	0.993	1.000	0.893	1.000
35	--	--	--	--	--	--	--	--	--	--	0.974	1.000
40	--	--	--	--	--	--	--	--	--	--	0.996	1.000
45	--	--	--	--	--	--	--	--	--	--	1.000	1.000
50	--	--	--	--	--	--	--	--	--	--	1.000	1.000

Average Run Length where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of $s = 0.5\,p$

# of Off-Target Streams	p = 5		10		15		20		30		50	
m_1^*	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
1	177.7	56.6	244.1	110.0	276.6	148.8	295.7	177.7	317.4	217.8	336.9	262.7
2	56.6	8.9	110.0	24.2	148.8	40.6	177.7	56.6	217.8	85.4	262.7	130.9
3	20.6	2.7	49.6	7.4	76.3	13.6	99.5	20.6	137.1	35.2	188.3	63.4
4	8.9	1.4	24.2	3.1	40.6	5.7	56.6	8.9	85.4	16.2	130.9	32.4
5	4.5	1.1	12.8	1.8	22.9	3.0	33.4	4.5	54.1	8.3	90.6	17.7
6	--	--	7.4	1.3	13.6	1.9	20.6	2.7	35.2	4.8	63.4	10.4
7	--	--	4.6	1.1	8.6	1.4	13.2	1.8	23.5	3.0	45.0	6.5
8	--	--	3.1	1.0	5.7	1.1	8.9	1.4	16.2	2.1	32.4	4.3
9	--	--	2.3	1.0	4.0	1.1	6.2	1.2	11.4	1.6	23.8	3.1
10	--	--	1.8	1.0	3.0	1.0	4.5	1.1	8.3	1.3	17.7	2.3
15	--	--	--	--	1.2	1.0	1.6	1.0	2.5	1.0	5.3	1.1
20	--	--	--	--	--	--	1.1	1.0	1.3	1.0	2.3	1.0
25	--	--	--	--	--	--	--	--	1.1	1.0	1.4	1.0
30	--	--	--	--	--	--	--	--	1.0	1.0	1.1	1.0
35	--	--	--	--	--	--	--	--	--	--	1.0	1.0
40	--	--	--	--	--	--	--	--	--	--	1.0	1.0
45	--	--	--	--	--	--	--	--	--	--	1.0	1.0
50	--	--	--	--	--	--	--	--	--	--	1.0	1.0

Probability of Detection where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of $s = p$

# of Off-Target Streams	p = 5		10		15		20		30		50	
m_1^*	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
1	0.006	0.024	0.004	0.010	0.004	0.007	0.003	0.006	0.003	0.005	0.003	0.004
2	0.013	0.078	0.008	0.036	0.006	0.021	0.005	0.015	0.004	0.010	0.004	0.007
3	0.024	0.164	0.014	0.086	0.009	0.049	0.008	0.034	0.006	0.021	0.004	0.012
4	0.038	0.282	0.024	0.165	0.015	0.094	0.012	0.067	0.008	0.039	0.006	0.020
5	0.056	0.432	0.039	0.271	0.023	0.158	0.018	0.115	0.012	0.066	0.008	0.033
6	--	--	0.060	0.399	0.034	0.239	0.026	0.180	0.016	0.105	0.010	0.051
7	--	--	0.087	0.543	0.049	0.336	0.037	0.259	0.023	0.154	0.013	0.076
8	--	--	0.123	0.689	0.069	0.444	0.051	0.351	0.030	0.216	0.017	0.108
9	--	--	0.168	0.824	0.093	0.556	0.069	0.450	0.040	0.286	0.022	0.147
10	--	--	0.222	0.930	0.122	0.666	0.091	0.552	0.053	0.365	0.027	0.193
15	--	--	--	--	0.362	0.989	0.274	0.930	0.158	0.761	0.077	0.500
20	--	--	--	--	--	--	0.564	1.000	0.346	0.963	0.173	0.794
25	--	--	--	--	--	--	--	--	0.587	0.999	0.319	0.948
30	--	--	--	--	--	--	--	--	0.809	1.000	0.500	0.993
35	--	--	--	--	--	--	--	--	--	--	0.683	1.000
40	--	--	--	--	--	--	--	--	--	--	0.832	1.000
45	--	--	--	--	--	--	--	--	--	--	0.929	1.000
50	--	--	--	--	--	--	--	--	--	--	0.977	1.000

Average Run Length where m^*_1 of p Streams are Off-Target
with a Fractional Sample Size of $s = p$

# of Off-Target Streams	p = 5		10		15		20		30		50	
m_1^*	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ	1σ	2σ
0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
1	165.8	41.4	240.2	98.2	274.0	137.5	294.3	169.6	316.7	212.3	336.6	259.8
2	76.5	12.8	130.7	28.2	174.0	47.5	202.1	65.7	241.3	99.3	282.5	150.7
3	42.0	6.1	71.5	11.6	106.7	20.5	131.4	29.0	173.0	48.3	224.7	84.8
4	26.1	3.5	41.5	6.1	66.7	10.7	85.3	14.9	121.5	25.8	173.5	49.3
5	17.7	2.3	25.7	3.7	43.2	6.3	56.6	8.7	85.6	15.1	132.3	30.2
6	--	--	16.8	2.5	29.1	4.2	38.6	5.6	61.1	9.6	100.8	19.5
7	--	--	11.5	1.8	20.3	3.0	27.1	3.9	44.4	6.5	77.2	13.2
8	--	--	8.1	1.5	14.6	2.3	19.6	2.8	32.8	4.6	59.6	9.3
9	--	--	6.0	1.2	10.8	1.8	14.5	2.2	24.7	3.5	46.5	6.8
10	--	--	4.5	1.1	8.2	1.5	11.0	1.8	18.9	2.7	36.6	5.2
15	--	--	--	--	2.8	1.0	3.6	1.1	6.3	1.3	13.0	2.0
20	--	--	--	--	--	--	1.8	1.0	2.9	1.0	5.8	1.3
25	--	--	--	--	--	--	--	--	1.7	1.0	3.1	1.1
30	--	--	--	--	--	--	--	--	1.2	1.0	2.0	1.0
35	--	--	--	--	--	--	--	--	--	--	1.5	1.0
40	--	--	--	--	--	--	--	--	--	--	1.2	1.0
45	--	--	--	--	--	--	--	--	--	--	1.1	1.0
50	--	--	--	--	--	--	--	--	--	--	1.0	1.0

Average Run Length Tables for $p = 20$ Streams

ARLs

Streams = 20
Off-Target = 20%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	289.99	148.89	64.42	29.51	15.66	9.79
10	370.38	281.06	138.85	60.45	29.12	16.49	10.66
15	370.38	272.46	129.19	55.26	26.30	14.71	9.52
20	370.38	264.31	120.58	50.68	23.86	13.19	8.42
25	370.38	256.61	112.94	46.71	21.78	11.94	7.56
30	370.38	249.32	106.16	43.26	20.00	10.88	6.85
35	370.38	242.42	100.11	40.26	18.47	9.98	6.25
40	370.38	235.88	94.68	37.62	17.14	9.20	5.73
45	370.38	229.67	89.78	35.29	15.97	8.52	5.28
50	370.38	223.77	85.34	33.21	14.94	7.93	4.89
55	370.38	218.16	81.30	31.35	14.02	7.40	4.55
60	370.38	212.81	77.61	29.67	13.20	6.93	4.24
65	370.38	207.72	74.22	28.16	12.46	6.51	3.97
70	370.38	202.85	71.10	26.78	11.80	6.14	3.73
75	370.38	198.20	68.22	25.52	11.19	5.80	3.50
80	370.38	193.75	65.55	24.36	10.64	5.49	3.30
85	370.38	189.49	63.07	23.30	10.14	5.21	3.12
90	370.38	185.41	60.76	22.32	9.68	4.95	2.95
95	370.38	181.49	58.61	21.41	9.25	4.71	2.80
100	370.38	177.73	56.59	20.56	8.86	4.50	2.66

ARLs

Streams = 20
Off-Target = 40%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	238.27	93.17	35.28	15.37	8.00	4.96
10	370.38	207.46	67.46	23.71	10.74	6.26	4.38
15	370.38	183.91	53.24	18.11	8.26	4.83	3.37
20	370.38	164.96	43.74	14.52	6.64	3.94	2.78
25	370.38	149.31	36.89	12.01	5.51	3.31	2.37
30	370.38	136.17	31.71	10.17	4.69	2.85	2.07
35	370.38	124.96	27.67	8.77	4.06	2.49	1.84
40	370.38	115.31	24.43	7.66	3.57	2.22	1.67
45	370.38	106.90	21.78	6.77	3.17	2.00	1.53
50	370.38	99.51	19.58	6.05	2.85	1.82	1.42
55	370.38	92.97	17.73	5.44	2.58	1.68	1.33
60	370.38	87.15	16.14	4.93	2.36	1.56	1.25
65	370.38	81.92	14.78	4.49	2.17	1.46	1.19
70	370.38	77.22	13.59	4.12	2.01	1.37	1.14
75	370.38	72.96	12.55	3.79	1.87	1.30	1.11
80	370.38	69.09	11.63	3.51	1.75	1.24	1.07
85	370.38	65.56	10.82	3.26	1.64	1.19	1.05
90	370.38	62.32	10.09	3.04	1.55	1.14	1.03
95	370.38	59.34	9.44	2.84	1.46	1.11	1.02
100	370.38	56.59	8.86	2.66	1.39	1.08	1.01

Average Run Length Tables for $p = 20$ Streams

ARLs — **Streams = 20**, **Off-Target = 60%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	202.21	67.80	24.29	10.39	5.37	3.32
10	370.38	154.16	39.12	12.56	5.59	3.33	2.46
15	370.38	123.50	26.83	8.42	3.94	2.48	1.87
20	370.38	102.15	19.96	6.23	3.02	1.98	1.56
25	370.38	86.44	15.61	4.88	2.45	1.68	1.36
30	370.38	74.45	12.64	3.98	2.07	1.47	1.24
35	370.38	65.00	10.50	3.34	1.80	1.33	1.15
40	370.38	57.40	8.90	2.88	1.61	1.23	1.10
45	370.38	51.16	7.67	2.52	1.46	1.16	1.06
50	370.38	45.96	6.69	2.24	1.35	1.11	1.03
55	370.38	41.57	5.91	2.02	1.26	1.07	1.02
60	370.38	37.82	5.27	1.84	1.20	1.04	1.01
65	370.38	34.58	4.73	1.70	1.15	1.03	1.00
70	370.38	31.77	4.29	1.58	1.11	1.02	1.00
75	370.38	29.31	3.91	1.48	1.08	1.01	1.00
80	370.38	27.14	3.58	1.40	1.05	1.00	1.00
85	370.38	25.21	3.30	1.33	1.04	1.00	1.00
90	370.38	23.49	3.06	1.27	1.02	1.00	1.00
95	370.38	21.95	2.85	1.22	1.01	1.00	1.00
100	370.38	20.56	2.66	1.18	1.01	1.00	1.00

ARLs — **Streams = 20**, **Off-Target = 80%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	175.63	53.29	18.52	7.85	4.04	2.50
10	370.38	116.86	25.37	7.76	3.43	2.08	1.59
15	370.38	85.14	15.46	4.69	2.28	1.56	1.28
20	370.38	65.55	10.64	3.30	1.75	1.30	1.13
25	370.38	52.39	7.89	2.55	1.46	1.16	1.06
30	370.38	43.03	6.15	2.08	1.29	1.08	1.03
35	370.38	36.09	4.98	1.77	1.18	1.04	1.01
40	370.38	30.78	4.14	1.56	1.11	1.02	1.00
45	370.38	26.61	3.53	1.41	1.07	1.01	1.00
50	370.38	23.26	3.07	1.30	1.04	1.00	1.00
55	370.38	20.53	2.71	1.22	1.02	1.00	1.00
60	370.38	18.28	2.43	1.16	1.01	1.00	1.00
65	370.38	16.39	2.20	1.12	1.01	1.00	1.00
70	370.38	14.79	2.01	1.09	1.00	1.00	1.00
75	370.38	13.42	1.86	1.06	1.00	1.00	1.00
80	370.38	12.25	1.73	1.04	1.00	1.00	1.00
85	370.38	11.23	1.62	1.03	1.00	1.00	1.00
90	370.38	10.33	1.53	1.02	1.00	1.00	1.00
95	370.38	9.55	1.46	1.01	1.00	1.00	1.00
100	370.38	8.86	1.39	1.01	1.00	1.00	1.00

Average Run Length Tables for $p = 40$ Streams

ARLs

Streams = 40
Off-Target = 20%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	279.43	135.83	58.32	27.99	15.94	10.41
10	370.38	260.10	114.25	46.76	21.97	12.31	7.99
15	370.38	243.08	98.02	38.53	17.79	9.88	6.39
20	370.38	228.00	85.49	32.52	14.81	8.18	5.29
25	370.38	214.56	75.55	27.97	12.60	6.93	4.49
30	370.38	202.51	67.49	24.41	10.89	5.98	3.88
35	370.38	191.64	60.83	21.56	9.54	5.23	3.41
40	370.38	181.80	55.23	19.23	8.45	4.63	3.03
45	370.38	172.83	50.47	17.29	7.56	4.14	2.72
50	370.38	164.65	46.38	15.66	6.81	3.74	2.47
55	370.38	157.13	42.82	14.27	6.18	3.39	2.25
60	370.38	150.22	39.70	13.07	5.63	3.10	2.07
65	370.38	143.83	36.95	12.03	5.17	2.85	1.92
70	370.38	137.92	34.51	11.12	4.76	2.63	1.79
75	370.38	132.43	32.32	10.31	4.40	2.44	1.67
80	370.38	127.31	30.36	9.60	4.09	2.27	1.57
85	370.38	122.54	28.58	8.96	3.81	2.13	1.48
90	370.38	118.08	26.97	8.39	3.56	1.99	1.40
95	370.38	113.90	25.51	7.87	3.34	1.88	1.33
100	370.38	109.97	24.17	7.40	3.13	1.77	1.27

ARLs

Streams = 40
Off-Target = 40%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	206.13	66.39	23.21	10.51	6.14	4.31
10	370.38	161.98	41.95	13.86	6.40	3.85	2.76
15	370.38	132.36	29.86	9.58	4.51	2.80	2.08
20	370.38	111.08	22.68	7.17	3.45	2.21	1.70
25	370.38	95.08	17.97	5.64	2.78	1.85	1.46
30	370.38	82.65	14.68	4.61	2.33	1.60	1.31
35	370.38	72.73	12.28	3.87	2.01	1.43	1.21
40	370.38	64.66	10.45	3.32	1.78	1.31	1.14
45	370.38	57.96	9.03	2.90	1.60	1.22	1.09
50	370.38	52.34	7.90	2.57	1.47	1.16	1.05
55	370.38	47.56	6.99	2.30	1.36	1.11	1.03
60	370.38	43.44	6.23	2.09	1.28	1.07	1.02
65	370.38	39.88	5.61	1.92	1.21	1.05	1.01
70	370.38	36.76	5.08	1.77	1.16	1.03	1.00
75	370.38	34.02	4.63	1.65	1.12	1.02	1.00
80	370.38	31.59	4.24	1.55	1.09	1.01	1.00
85	370.38	29.43	3.91	1.46	1.06	1.01	1.00
90	370.38	27.49	3.61	1.39	1.05	1.00	1.00
95	370.38	25.75	3.36	1.33	1.03	1.00	1.00
100	370.38	24.17	3.13	1.27	1.02	1.00	1.00

Average Run Length Tables for p = 40 Streams

ARLs

Streams = 40
Off-Target = 60%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	153.43	38.76	12.42	5.52	3.30	2.44
10	370.38	100.77	19.46	6.08	2.98	1.98	1.57
15	370.38	72.90	12.21	3.89	2.06	1.49	1.26
20	370.38	55.85	8.56	2.83	1.62	1.26	1.12
25	370.38	44.47	6.43	2.23	1.38	1.13	1.05
30	370.38	36.41	5.06	1.85	1.23	1.07	1.02
35	370.38	30.46	4.14	1.60	1.14	1.03	1.01
40	370.38	25.93	3.47	1.43	1.08	1.02	1.00
45	370.38	22.37	2.99	1.31	1.05	1.01	1.00
50	370.38	19.53	2.61	1.22	1.03	1.00	1.00
55	370.38	17.22	2.33	1.16	1.01	1.00	1.00
60	370.38	15.31	2.10	1.11	1.01	1.00	1.00
65	370.38	13.72	1.91	1.08	1.00	1.00	1.00
70	370.38	12.37	1.76	1.05	1.00	1.00	1.00
75	370.38	11.23	1.64	1.04	1.00	1.00	1.00
80	370.38	10.24	1.54	1.02	1.00	1.00	1.00
85	370.38	9.38	1.45	1.02	1.00	1.00	1.00
90	370.38	8.64	1.38	1.01	1.00	1.00	1.00
95	370.38	7.98	1.32	1.01	1.00	1.00	1.00
100	370.38	7.40	1.27	1.00	1.00	1.00	1.00

ARLs

Streams = 40
Off-Target = 80%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	116.58	25.26	7.73	3.41	2.07	1.59
10	370.38	65.10	10.52	3.27	1.75	1.30	1.14
15	370.38	42.57	6.06	2.07	1.30	1.09	1.03
20	370.38	30.36	4.09	1.57	1.12	1.03	1.01
25	370.38	22.89	3.04	1.32	1.05	1.01	1.00
30	370.38	17.95	2.41	1.18	1.02	1.00	1.00
35	370.38	14.51	2.01	1.10	1.01	1.00	1.00
40	370.38	12.00	1.74	1.05	1.00	1.00	1.00
45	370.38	10.12	1.55	1.03	1.00	1.00	1.00
50	370.38	8.67	1.41	1.02	1.00	1.00	1.00
55	370.38	7.53	1.31	1.01	1.00	1.00	1.00
60	370.38	6.62	1.23	1.00	1.00	1.00	1.00
65	370.38	5.88	1.17	1.00	1.00	1.00	1.00
70	370.38	5.26	1.13	1.00	1.00	1.00	1.00
75	370.38	4.75	1.10	1.00	1.00	1.00	1.00
80	370.38	4.32	1.07	1.00	1.00	1.00	1.00
85	370.38	3.96	1.05	1.00	1.00	1.00	1.00
90	370.38	3.64	1.04	1.00	1.00	1.00	1.00
95	370.38	3.37	1.03	1.00	1.00	1.00	1.00
100	370.38	3.13	1.02	1.00	1.00	1.00	1.00

Average Run Length Tables for p = 60 Streams

ARLs — **Streams = 60**, **Off-Target = 20%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	268.50	122.71	51.09	24.24	13.70	9.01
10	370.38	241.08	95.54	37.17	17.19	9.61	6.27
15	370.38	218.36	77.42	28.68	13.03	7.26	4.75
20	370.38	199.24	64.54	23.03	10.34	5.76	3.80
25	370.38	182.93	54.96	19.04	8.47	4.74	3.15
30	370.38	168.88	47.58	16.08	7.12	4.00	2.69
35	370.38	156.63	41.72	13.81	6.09	3.44	2.35
40	370.38	145.88	36.98	12.03	5.29	3.02	2.08
45	370.38	136.37	33.08	10.59	4.66	2.68	1.87
50	370.38	127.89	29.80	9.42	4.15	2.40	1.71
55	370.38	120.29	27.03	8.44	3.72	2.18	1.57
60	370.38	113.45	24.66	7.62	3.37	2.00	1.46
65	370.38	107.25	22.60	6.92	3.07	1.84	1.37
70	370.38	101.62	20.81	6.32	2.81	1.71	1.30
75	370.38	96.47	19.24	5.80	2.60	1.60	1.23
80	370.38	91.76	17.85	5.35	2.41	1.50	1.18
85	370.38	87.42	16.61	4.95	2.24	1.42	1.14
90	370.38	83.42	15.51	4.60	2.10	1.35	1.10
95	370.38	79.72	14.52	4.29	1.97	1.29	1.08
100	370.38	76.29	13.62	4.01	1.85	1.24	1.05

ARLs — **Streams = 60**, **Off-Target = 40%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	180.89	51.17	17.27	7.92	4.70	3.32
10	370.38	131.17	29.30	9.41	4.46	2.79	2.08
15	370.38	101.20	19.61	6.20	3.05	2.01	1.58
20	370.38	81.27	14.27	4.52	2.32	1.61	1.32
25	370.38	67.14	10.96	3.52	1.89	1.38	1.19
30	370.38	56.65	8.75	2.86	1.62	1.24	1.10
35	370.38	48.60	7.18	2.41	1.43	1.15	1.06
40	370.38	42.25	6.04	2.09	1.30	1.09	1.03
45	370.38	37.14	5.17	1.85	1.21	1.06	1.01
50	370.38	32.95	4.49	1.67	1.15	1.03	1.01
55	370.38	29.47	3.96	1.53	1.10	1.02	1.00
60	370.38	26.54	3.53	1.42	1.07	1.01	1.00
65	370.38	24.04	3.17	1.33	1.04	1.00	1.00
70	370.38	21.90	2.88	1.26	1.03	1.00	1.00
75	370.38	20.04	2.64	1.20	1.02	1.00	1.00
80	370.38	18.42	2.43	1.16	1.01	1.00	1.00
85	370.38	16.99	2.25	1.12	1.01	1.00	1.00
90	370.38	15.74	2.10	1.09	1.00	1.00	1.00
95	370.38	14.62	1.97	1.07	1.00	1.00	1.00
100	370.38	13.62	1.85	1.05	1.00	1.00	1.00

Average Run Length Tables for p = 60 Streams

ARLs — Streams = 60, Off-Target = 60%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	121.99	26.19	8.20	3.85	2.45	1.87
10	370.38	72.42	12.08	3.86	2.06	1.49	1.26
15	370.38	49.16	7.28	2.48	1.49	1.19	1.09
20	370.38	35.99	5.01	1.85	1.24	1.08	1.03
25	370.38	27.67	3.74	1.52	1.12	1.03	1.01
30	370.38	22.04	2.96	1.32	1.06	1.01	1.00
35	370.38	18.03	2.45	1.20	1.03	1.00	1.00
40	370.38	15.06	2.09	1.12	1.01	1.00	1.00
45	370.38	12.79	1.84	1.08	1.00	1.00	1.00
50	370.38	11.03	1.65	1.05	1.00	1.00	1.00
55	370.38	9.62	1.51	1.03	1.00	1.00	1.00
60	370.38	8.48	1.40	1.02	1.00	1.00	1.00
65	370.38	7.55	1.31	1.01	1.00	1.00	1.00
70	370.38	6.77	1.24	1.00	1.00	1.00	1.00
75	370.38	6.12	1.19	1.00	1.00	1.00	1.00
80	370.38	5.56	1.15	1.00	1.00	1.00	1.00
85	370.38	5.09	1.12	1.00	1.00	1.00	1.00
90	370.38	4.68	1.09	1.00	1.00	1.00	1.00
95	370.38	4.32	1.07	1.00	1.00	1.00	1.00
100	370.38	4.01	1.05	1.00	1.00	1.00	1.00

ARLs — Streams = 60, Off-Target = 80%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	84.61	15.30	4.63	2.26	1.55	1.29
10	370.38	42.43	6.04	2.07	1.30	1.10	1.04
15	370.38	26.08	3.48	1.43	1.08	1.02	1.00
20	370.38	17.85	2.41	1.18	1.02	1.00	1.00
25	370.38	13.08	1.86	1.08	1.00	1.00	1.00
30	370.38	10.06	1.55	1.03	1.00	1.00	1.00
35	370.38	8.02	1.36	1.01	1.00	1.00	1.00
40	370.38	6.58	1.23	1.00	1.00	1.00	1.00
45	370.38	5.52	1.15	1.00	1.00	1.00	1.00
50	370.38	4.73	1.10	1.00	1.00	1.00	1.00
55	370.38	4.11	1.07	1.00	1.00	1.00	1.00
60	370.38	3.63	1.04	1.00	1.00	1.00	1.00
65	370.38	3.24	1.03	1.00	1.00	1.00	1.00
70	370.38	2.92	1.02	1.00	1.00	1.00	1.00
75	370.38	2.66	1.01	1.00	1.00	1.00	1.00
80	370.38	2.44	1.01	1.00	1.00	1.00	1.00
85	370.38	2.26	1.00	1.00	1.00	1.00	1.00
90	370.38	2.10	1.00	1.00	1.00	1.00	1.00
95	370.38	1.97	1.00	1.00	1.00	1.00	1.00
100	370.38	1.85	1.00	1.00	1.00	1.00	1.00

Average Run Length Tables for p = 80 Streams

ARLs — **Streams = 80**, **Off-Target = 20%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	258.10	111.34	45.04	21.16	11.93	7.81
10	370.38	224.25	81.42	30.45	13.92	7.80	5.13
15	370.38	197.63	63.14	22.41	10.09	5.67	3.77
20	370.38	176.18	50.92	17.39	7.77	4.40	2.97
25	370.38	158.53	42.21	14.00	6.23	3.56	2.44
30	370.38	143.77	35.73	11.58	5.15	2.98	2.09
35	370.38	131.26	30.74	9.78	4.36	2.56	1.83
40	370.38	120.52	26.80	8.40	3.77	2.25	1.63
45	370.38	111.21	23.62	7.31	3.30	2.00	1.49
50	370.38	103.07	21.01	6.44	2.93	1.81	1.38
55	370.38	95.90	18.83	5.73	2.63	1.66	1.29
60	370.38	89.53	17.00	5.14	2.38	1.53	1.22
65	370.38	83.84	15.43	4.64	2.18	1.43	1.16
70	370.38	78.74	14.08	4.22	2.01	1.35	1.12
75	370.38	74.13	12.91	3.86	1.86	1.28	1.09
80	370.38	69.96	11.89	3.55	1.74	1.22	1.06
85	370.38	66.16	10.99	3.29	1.63	1.17	1.04
90	370.38	62.70	10.20	3.05	1.54	1.13	1.03
95	370.38	59.52	9.49	2.84	1.46	1.10	1.02
100	370.38	56.59	8.86	2.66	1.39	1.08	1.01

ARLs — **Streams = 80**, **Off-Target = 40%**

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	160.58	41.13	13.56	6.30	3.81	2.75
10	370.38	109.12	21.92	6.96	3.40	2.21	1.71
15	370.38	80.60	14.07	4.48	2.32	1.62	1.33
20	370.38	62.66	9.98	3.25	1.79	1.34	1.16
25	370.38	50.45	7.53	2.54	1.50	1.19	1.08
30	370.38	41.68	5.95	2.09	1.32	1.10	1.04
35	370.38	35.13	4.86	1.79	1.20	1.05	1.02
40	370.38	30.08	4.07	1.58	1.13	1.03	1.01
45	370.38	26.09	3.49	1.43	1.08	1.01	1.00
50	370.38	22.88	3.04	1.32	1.05	1.01	1.00
55	370.38	20.25	2.70	1.23	1.03	1.00	1.00
60	370.38	18.07	2.42	1.17	1.02	1.00	1.00
65	370.38	16.23	2.19	1.13	1.01	1.00	1.00
70	370.38	14.68	2.01	1.09	1.00	1.00	1.00
75	370.38	13.34	1.86	1.07	1.00	1.00	1.00
80	370.38	12.19	1.73	1.05	1.00	1.00	1.00
85	370.38	11.19	1.63	1.03	1.00	1.00	1.00
90	370.38	10.31	1.53	1.02	1.00	1.00	1.00
95	370.38	9.54	1.46	1.01	1.00	1.00	1.00
100	370.38	8.86	1.39	1.01	1.00	1.00	1.00

Average Run Length Tables for p = 80 Streams

ARLs — **Streams = 80**, **Off-Target = 60%**

	shift						
% sampled	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
5	370.38	100.13	19.23	6.01	2.96	1.98	1.57
10	370.38	55.13	8.41	2.80	1.63	1.27	1.13
15	370.38	35.78	4.98	1.85	1.24	1.08	1.03
20	370.38	25.39	3.43	1.45	1.10	1.02	1.01
25	370.38	19.08	2.59	1.24	1.04	1.01	1.00
30	370.38	14.93	2.09	1.13	1.01	1.00	1.00
35	370.38	12.05	1.77	1.07	1.00	1.00	1.00
40	370.38	9.97	1.55	1.04	1.00	1.00	1.00
45	370.38	8.41	1.40	1.02	1.00	1.00	1.00
50	370.38	7.21	1.29	1.01	1.00	1.00	1.00
55	370.38	6.27	1.21	1.00	1.00	1.00	1.00
60	370.38	5.52	1.16	1.00	1.00	1.00	1.00
65	370.38	4.91	1.11	1.00	1.00	1.00	1.00
70	370.38	4.40	1.08	1.00	1.00	1.00	1.00
75	370.38	3.98	1.06	1.00	1.00	1.00	1.00
80	370.38	3.63	1.04	1.00	1.00	1.00	1.00
85	370.38	3.33	1.03	1.00	1.00	1.00	1.00
90	370.38	3.07	1.02	1.00	1.00	1.00	1.00
95	370.38	2.85	1.01	1.00	1.00	1.00	1.00
100	370.38	2.66	1.01	1.00	1.00	1.00	1.00

ARLs — **Streams = 80**, **Off-Target = 80%**

	shift						
% sampled	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
5	370.38	64.89	10.47	3.26	1.74	1.30	1.15
10	370.38	30.16	4.06	1.57	1.13	1.03	1.01
15	370.38	17.80	2.40	1.19	1.02	1.00	1.00
20	370.38	11.89	1.74	1.06	1.00	1.00	1.00
25	370.38	8.59	1.41	1.02	1.00	1.00	1.00
30	370.38	6.56	1.24	1.01	1.00	1.00	1.00
35	370.38	5.22	1.14	1.00	1.00	1.00	1.00
40	370.38	4.29	1.08	1.00	1.00	1.00	1.00
45	370.38	3.62	1.04	1.00	1.00	1.00	1.00
50	370.38	3.12	1.02	1.00	1.00	1.00	1.00
55	370.38	2.74	1.01	1.00	1.00	1.00	1.00
60	370.38	2.44	1.01	1.00	1.00	1.00	1.00
65	370.38	2.20	1.00	1.00	1.00	1.00	1.00
70	370.38	2.01	1.00	1.00	1.00	1.00	1.00
75	370.38	1.86	1.00	1.00	1.00	1.00	1.00
80	370.38	1.73	1.00	1.00	1.00	1.00	1.00
85	370.38	1.62	1.00	1.00	1.00	1.00	1.00
90	370.38	1.53	1.00	1.00	1.00	1.00	1.00
95	370.38	1.46	1.00	1.00	1.00	1.00	1.00
100	370.38	1.39	1.00	1.00	1.00	1.00	1.00

Average Run Length Tables for p = 100 Streams

ARLs

Streams = 100
Off-Target = 20%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	248.31	101.59	40.05	18.67	10.52	6.89
10	370.38	209.33	70.48	25.53	11.59	6.52	4.33
15	370.38	180.08	52.76	18.12	8.14	4.62	3.12
20	370.38	157.34	41.45	13.72	6.14	3.54	2.44
25	370.38	139.19	33.66	10.84	4.86	2.85	2.02
30	370.38	124.39	28.01	8.84	3.99	2.39	1.73
35	370.38	112.10	23.76	7.39	3.37	2.06	1.54
40	370.38	101.75	20.46	6.29	2.90	1.82	1.39
45	370.38	92.92	17.84	5.44	2.55	1.64	1.29
50	370.38	85.31	15.72	4.77	2.27	1.50	1.21
55	370.38	78.68	13.98	4.23	2.05	1.39	1.15
60	370.38	72.87	12.53	3.79	1.87	1.30	1.11
65	370.38	67.74	11.30	3.42	1.72	1.23	1.07
70	370.38	63.18	10.25	3.11	1.60	1.18	1.05
75	370.38	59.10	9.35	2.85	1.50	1.13	1.03
80	370.38	55.43	8.58	2.63	1.41	1.10	1.02
85	370.38	52.12	7.90	2.44	1.34	1.07	1.01
90	370.38	49.12	7.30	2.27	1.28	1.05	1.01
95	370.38	46.39	6.77	2.13	1.23	1.04	1.00
100	370.38	43.89	6.30	2.00	1.19	1.02	1.00

ARLs

Streams = 100
Off-Target = 40%

% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	143.91	34.03	11.04	5.19	3.21	2.36
10	370.38	92.66	17.16	5.45	2.75	1.86	1.48
15	370.38	66.09	10.71	3.48	1.90	1.40	1.20
20	370.38	50.09	7.47	2.53	1.50	1.19	1.08
25	370.38	39.55	5.59	2.00	1.29	1.09	1.03
30	370.38	32.15	4.40	1.68	1.17	1.04	1.01
35	370.38	26.74	3.60	1.47	1.10	1.02	1.00
40	370.38	22.64	3.03	1.32	1.05	1.01	1.00
45	370.38	19.45	2.61	1.23	1.03	1.00	1.00
50	370.38	16.92	2.30	1.16	1.01	1.00	1.00
55	370.38	14.87	2.05	1.11	1.01	1.00	1.00
60	370.38	13.18	1.86	1.07	1.00	1.00	1.00
65	370.38	11.78	1.71	1.05	1.00	1.00	1.00
70	370.38	10.61	1.58	1.03	1.00	1.00	1.00
75	370.38	9.60	1.48	1.02	1.00	1.00	1.00
80	370.38	8.75	1.40	1.01	1.00	1.00	1.00
85	370.38	8.01	1.33	1.01	1.00	1.00	1.00
90	370.38	7.36	1.28	1.00	1.00	1.00	1.00
95	370.38	6.80	1.23	1.00	1.00	1.00	1.00
100	370.38	6.30	1.19	1.00	1.00	1.00	1.00

Average Run Length Tables for p = 100 Streams

ARLs

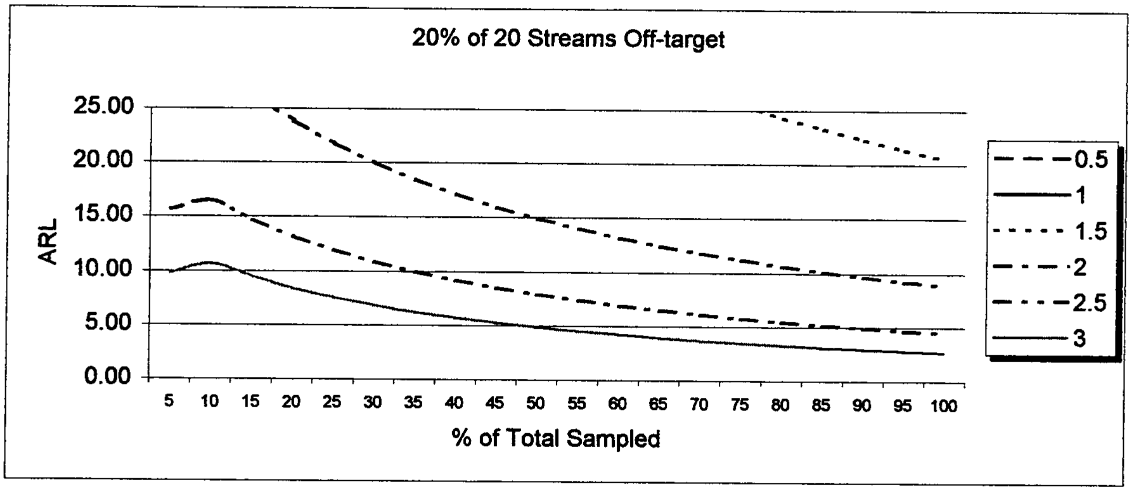
Streams = 100
Off-Target = 60%

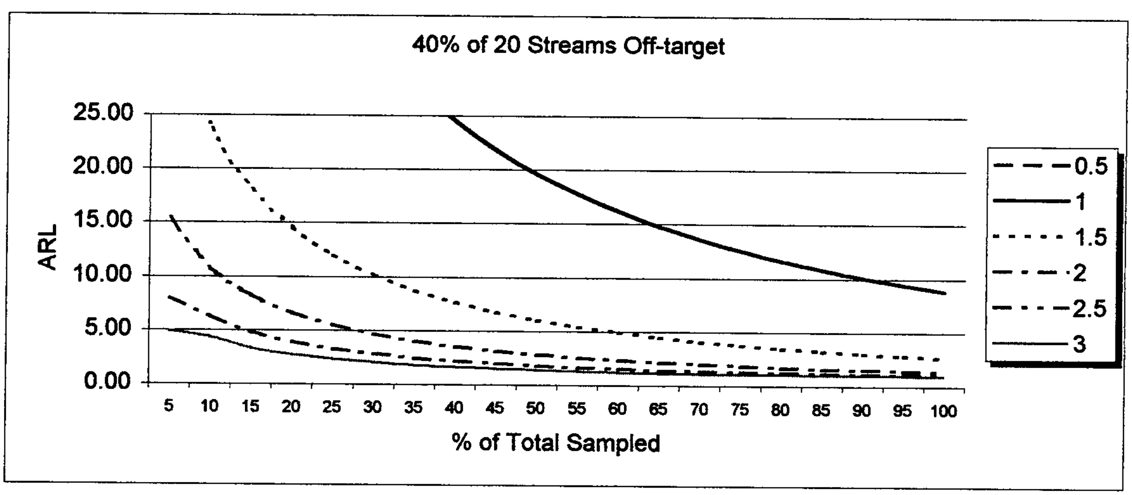
% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	84.12	14.89	4.69	2.41	1.69	1.39
10	370.38	43.66	6.29	2.22	1.39	1.15	1.06
15	370.38	27.38	3.71	1.52	1.13	1.03	1.01
20	370.38	18.99	2.59	1.24	1.04	1.01	1.00
25	370.38	14.05	2.00	1.12	1.01	1.00	1.00
30	370.38	10.88	1.65	1.05	1.00	1.00	1.00
35	370.38	8.72	1.44	1.02	1.00	1.00	1.00
40	370.38	7.17	1.30	1.01	1.00	1.00	1.00
45	370.38	6.04	1.20	1.00	1.00	1.00	1.00
50	370.38	5.17	1.14	1.00	1.00	1.00	1.00
55	370.38	4.50	1.09	1.00	1.00	1.00	1.00
60	370.38	3.97	1.06	1.00	1.00	1.00	1.00
65	370.38	3.54	1.04	1.00	1.00	1.00	1.00
70	370.38	3.19	1.03	1.00	1.00	1.00	1.00
75	370.38	2.90	1.02	1.00	1.00	1.00	1.00
80	370.38	2.66	1.01	1.00	1.00	1.00	1.00
85	370.38	2.45	1.01	1.00	1.00	1.00	1.00
90	370.38	2.28	1.00	1.00	1.00	1.00	1.00
95	370.38	2.13	1.00	1.00	1.00	1.00	1.00
100	370.38	2.00	1.00	1.00	1.00	1.00	1.00

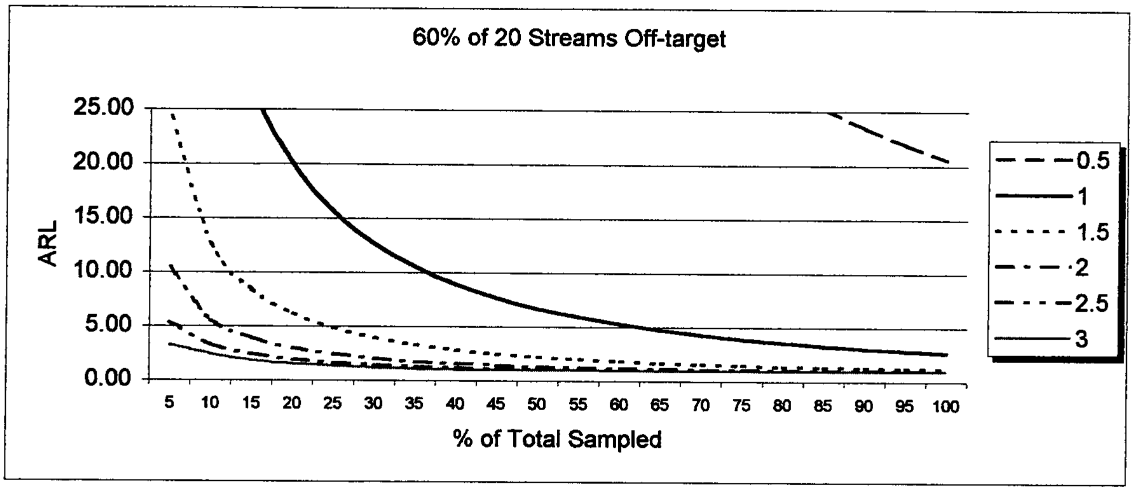
ARLs

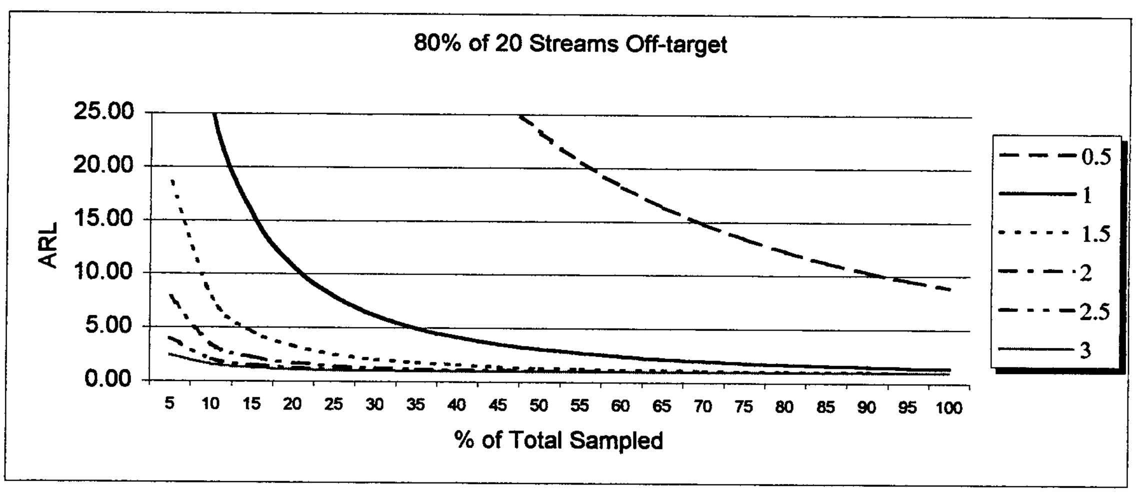
Streams = 100
Off-Target = 80%

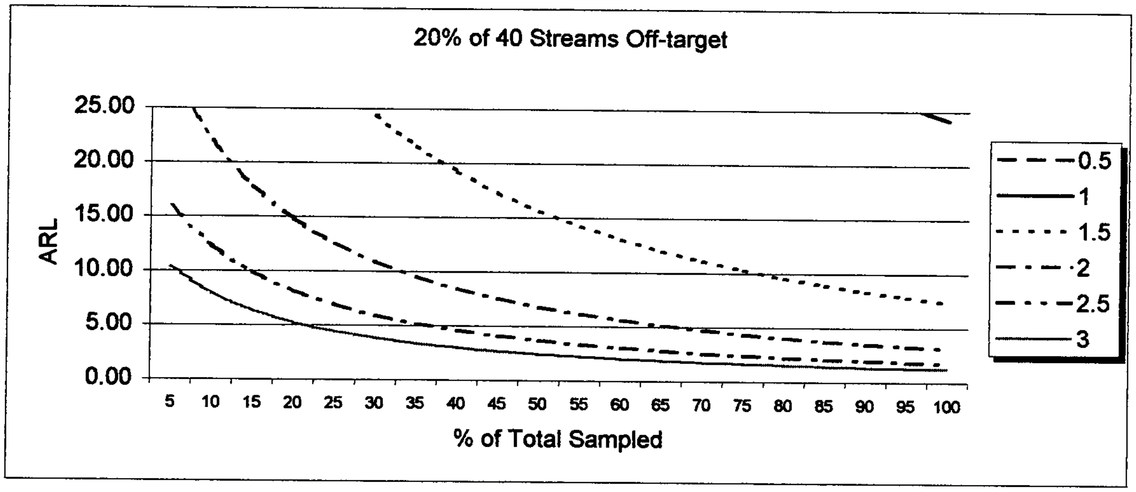
% sampled	shift 0	0.5	1	1.5	2	2.5	3
5	370.38	51.67	7.73	2.52	1.47	1.17	1.08
10	370.38	22.68	3.02	1.32	1.06	1.01	1.00
15	370.38	13.02	1.86	1.08	1.01	1.00	1.00
20	370.38	8.58	1.41	1.02	1.00	1.00	1.00
25	370.38	6.16	1.21	1.00	1.00	1.00	1.00
30	370.38	4.71	1.11	1.00	1.00	1.00	1.00
35	370.38	3.76	1.05	1.00	1.00	1.00	1.00
40	370.38	3.11	1.03	1.00	1.00	1.00	1.00
45	370.38	2.65	1.01	1.00	1.00	1.00	1.00
50	370.38	2.31	1.01	1.00	1.00	1.00	1.00
55	370.38	2.05	1.00	1.00	1.00	1.00	1.00
60	370.38	1.86	1.00	1.00	1.00	1.00	1.00
65	370.38	1.70	1.00	1.00	1.00	1.00	1.00
70	370.38	1.58	1.00	1.00	1.00	1.00	1.00
75	370.38	1.48	1.00	1.00	1.00	1.00	1.00
80	370.38	1.39	1.00	1.00	1.00	1.00	1.00
85	370.38	1.33	1.00	1.00	1.00	1.00	1.00
90	370.38	1.27	1.00	1.00	1.00	1.00	1.00
95	370.38	1.23	1.00	1.00	1.00	1.00	1.00
100	370.38	1.19	1.00	1.00	1.00	1.00	1.00

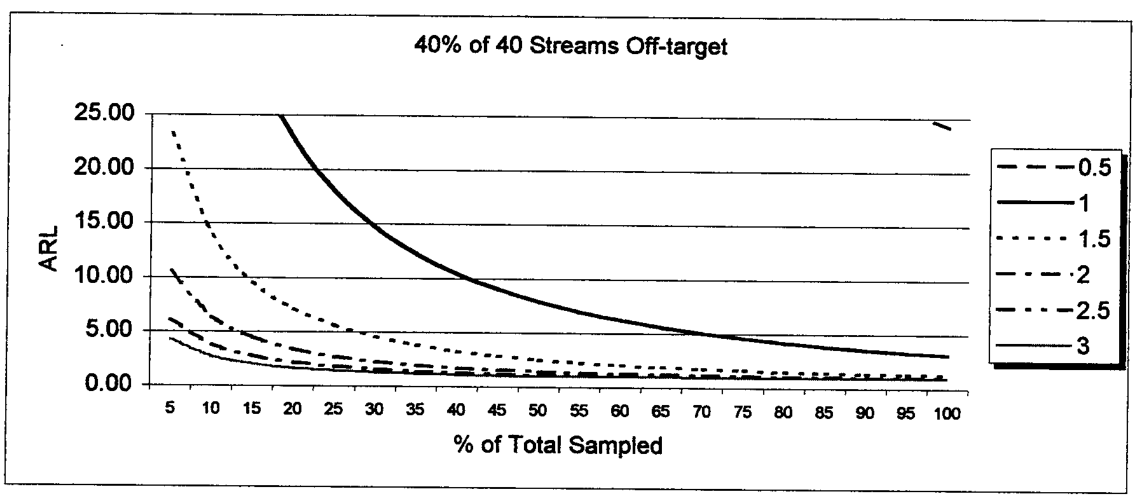

20% of 20 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

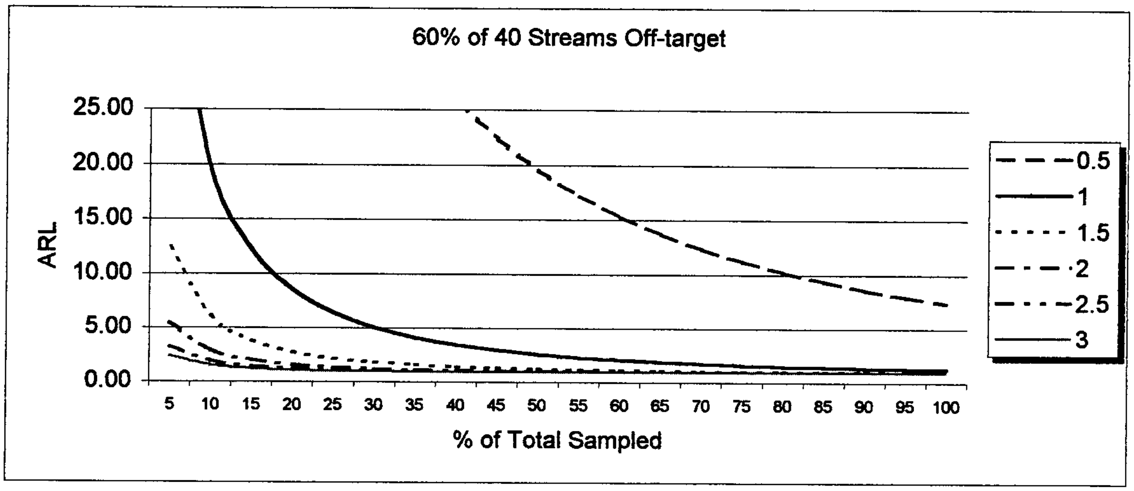

40% of 20 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

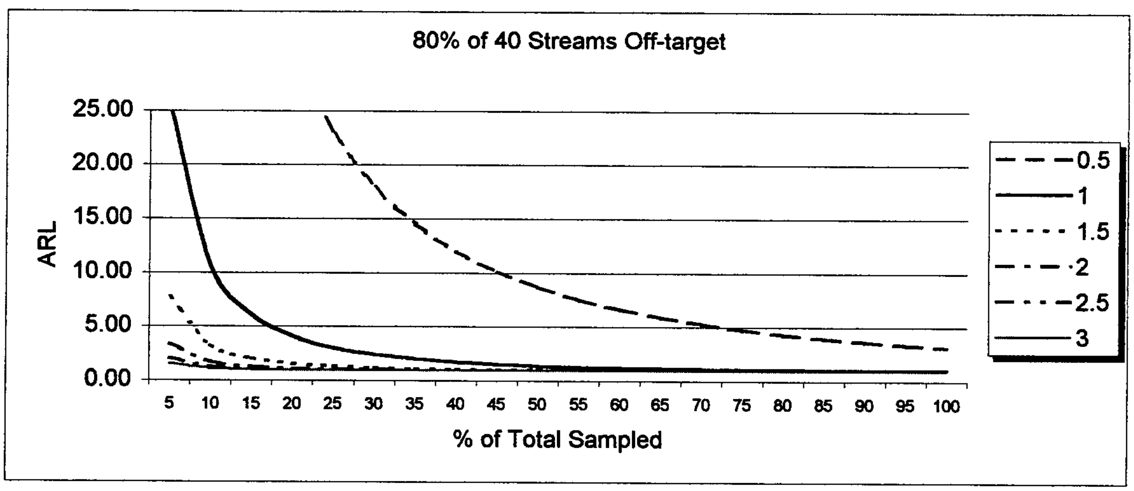

60% of 20 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

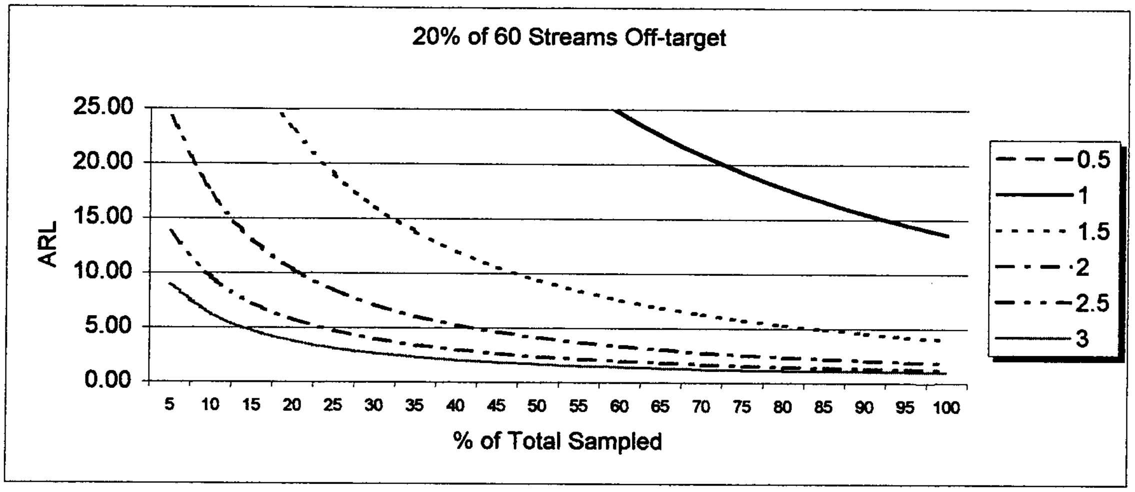

80% of 20 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

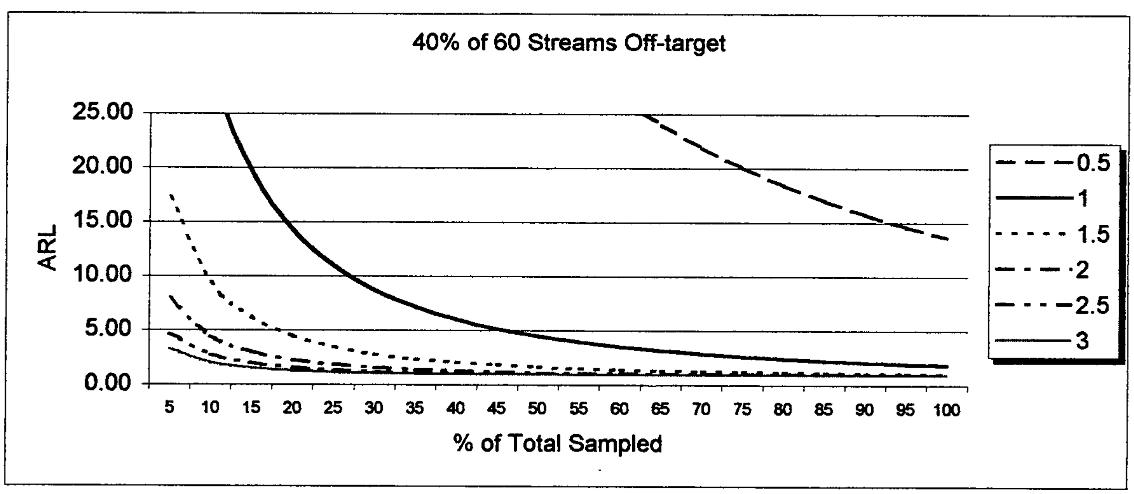

20% of 40 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

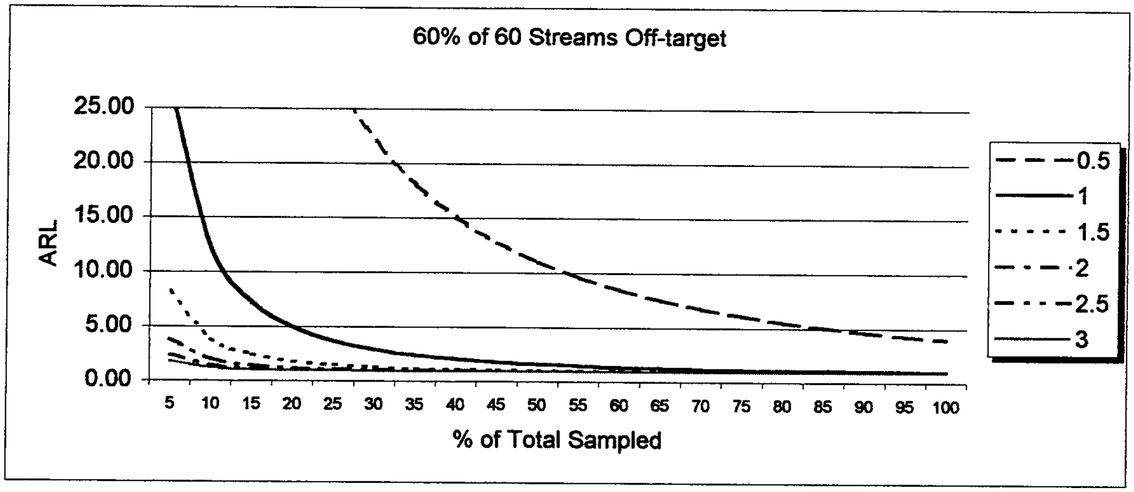

40% of 40 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


60% of 40 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

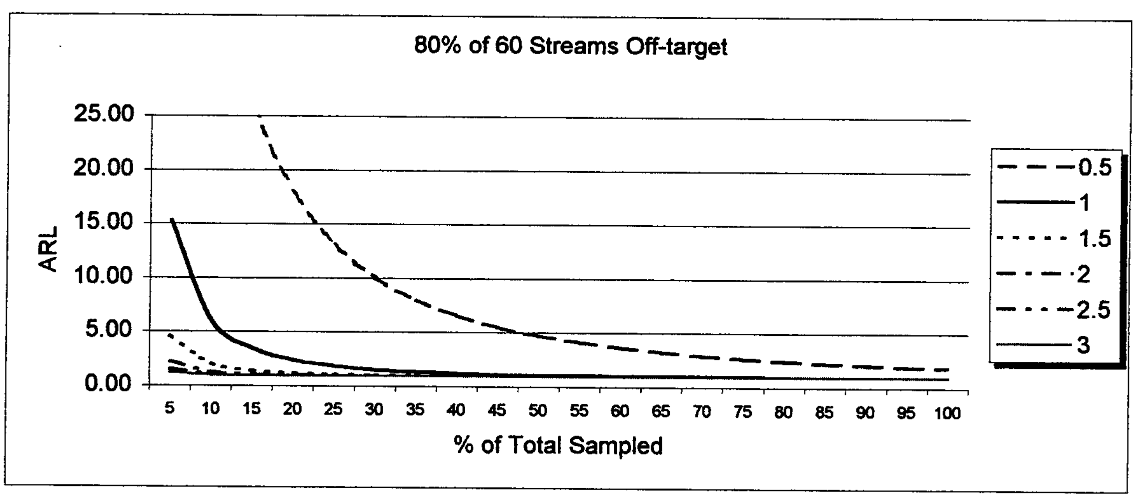

80% of 40 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


20% of 60 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

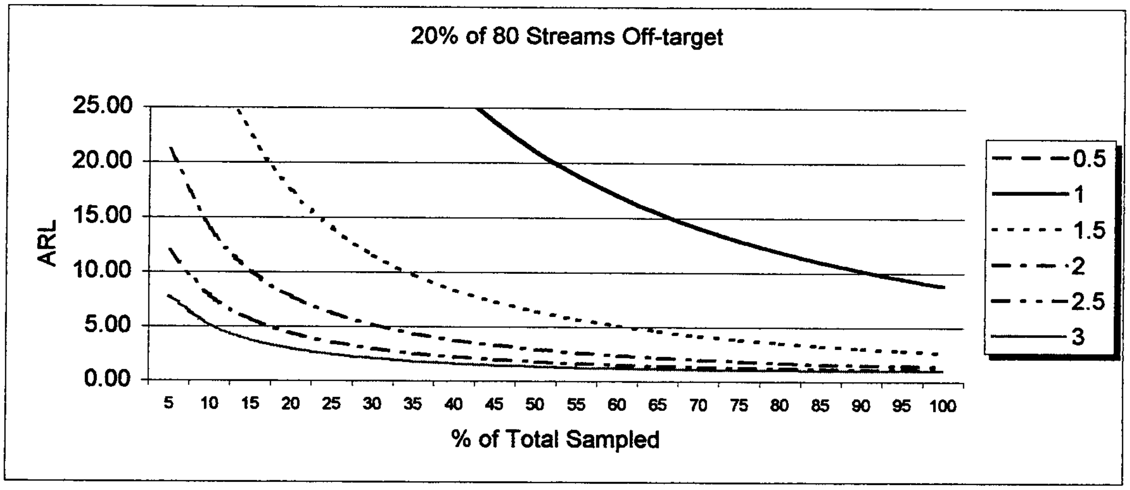

40% of 60 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

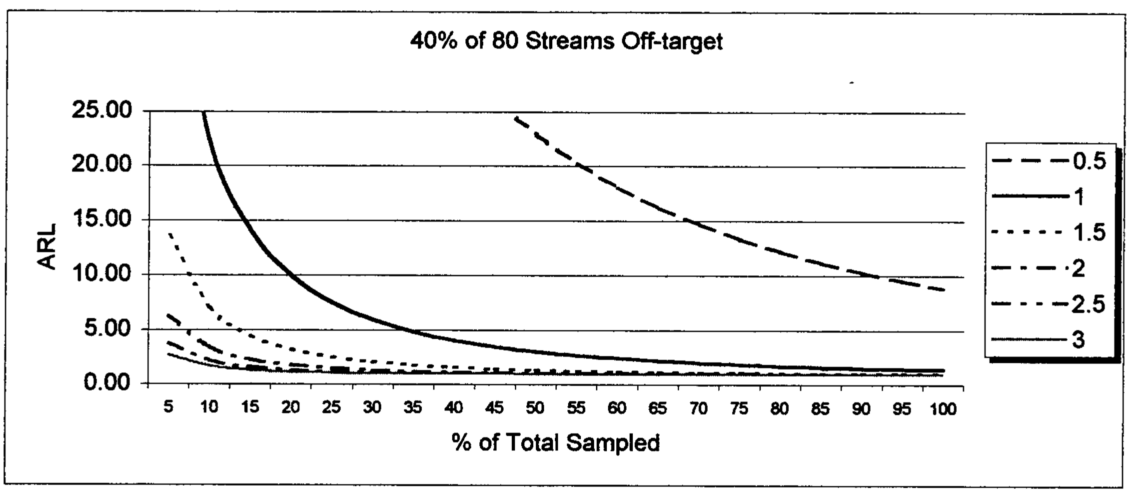


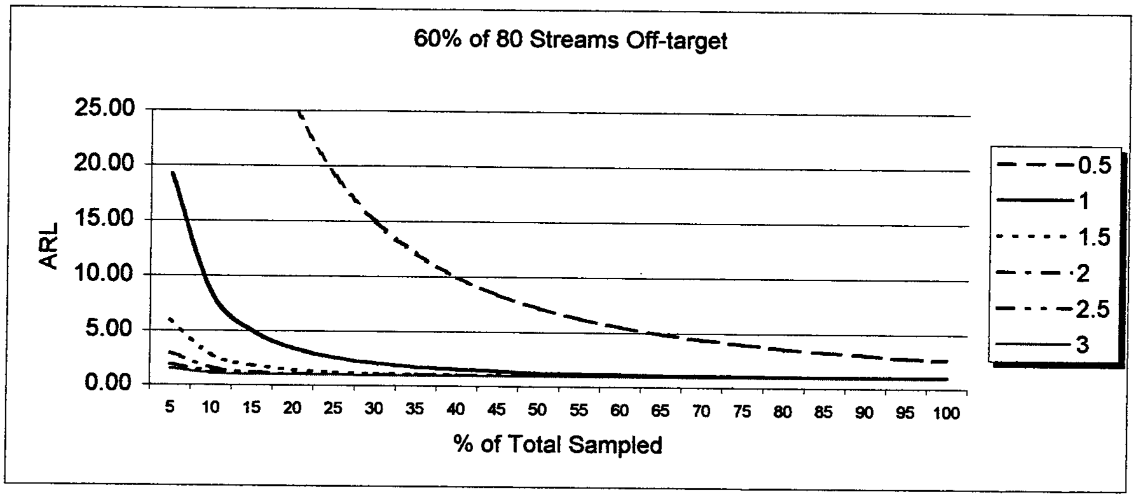
60% of 60 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

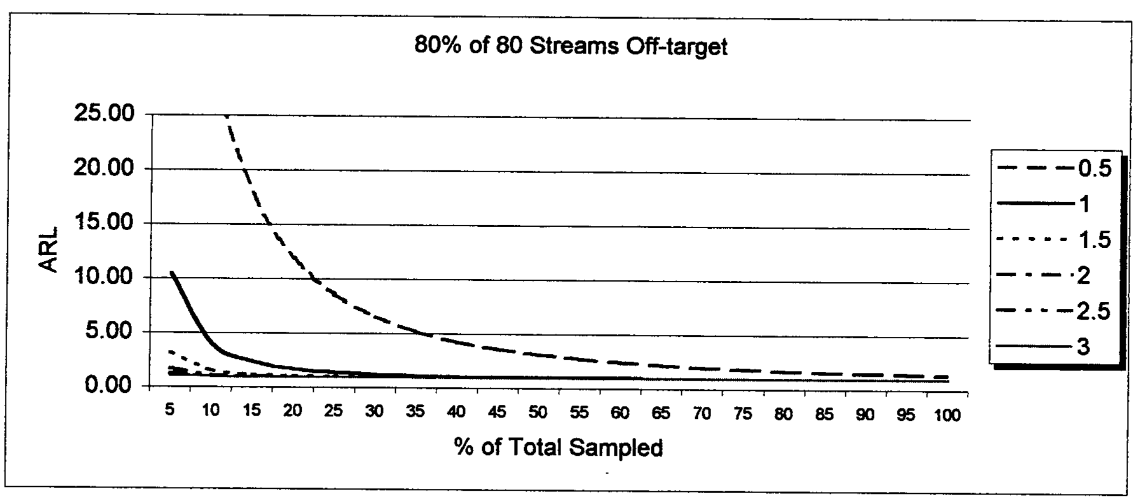


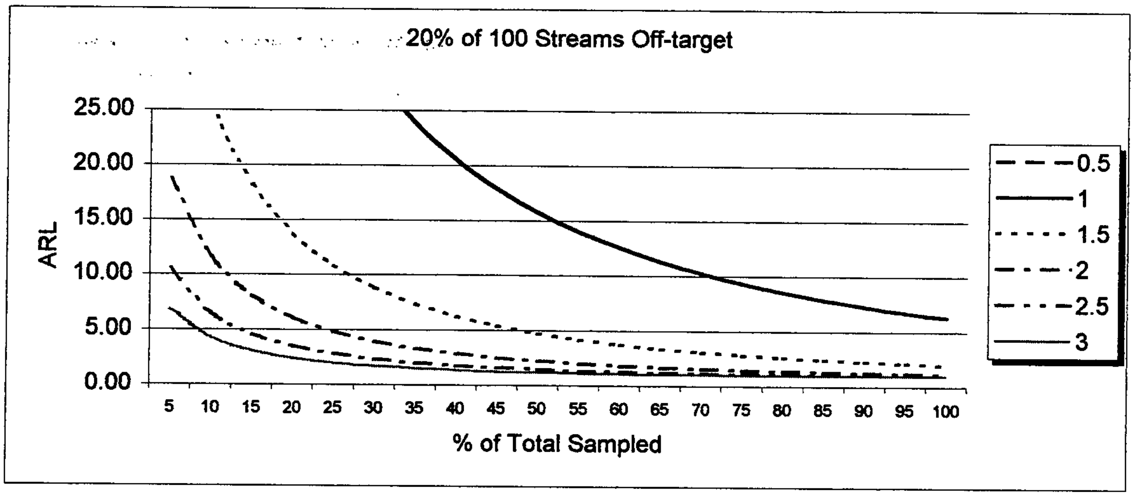
80% of 60 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

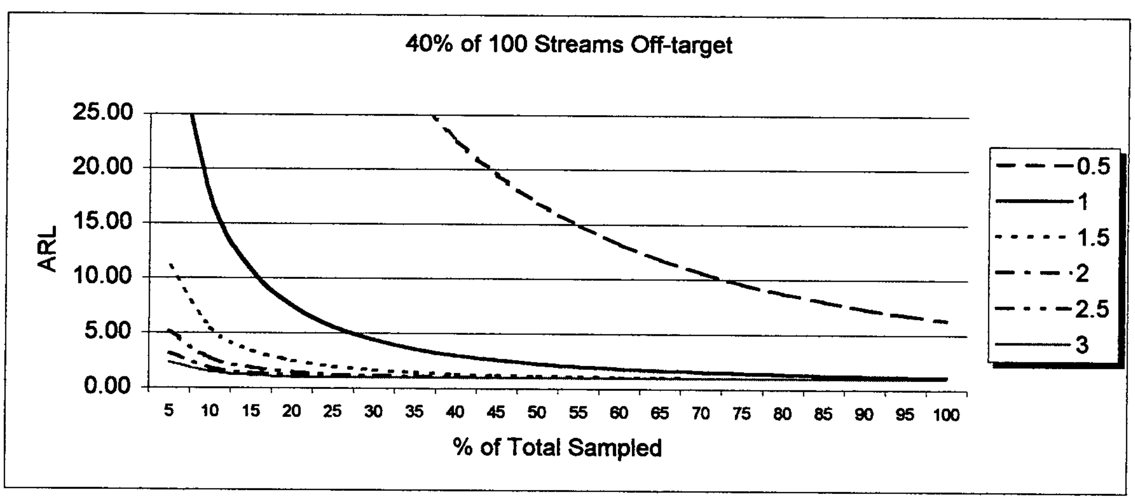

20% of 80 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

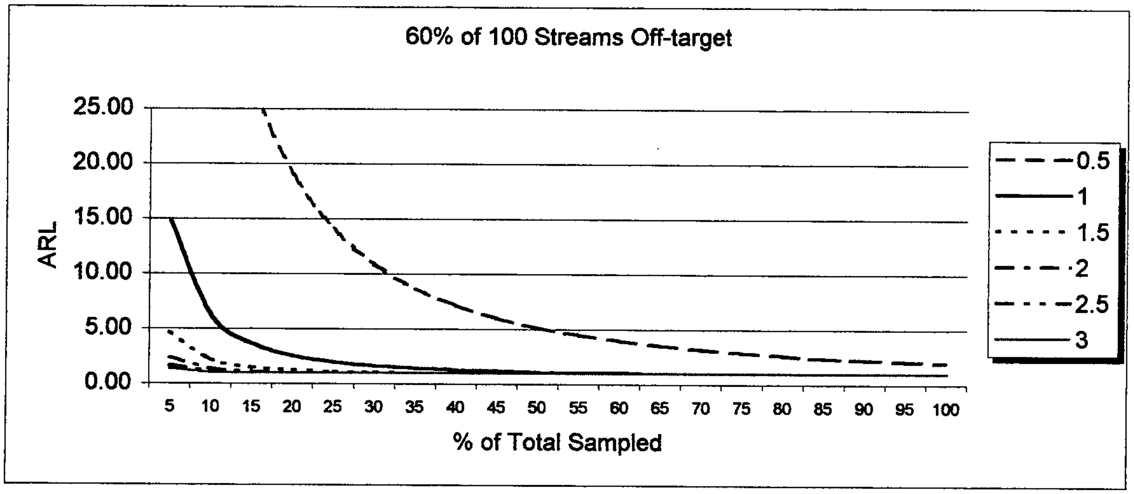

40% of 80 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

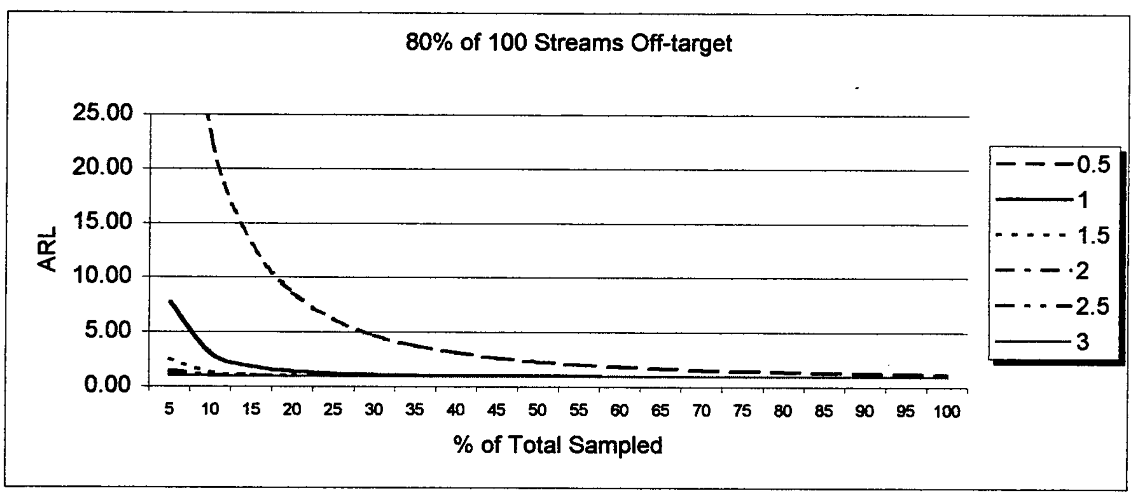

60% of 80 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


80% of 80 Streams Off-target
25.00
20.00
15.00
10.00
5.00
0.00
ARL
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


20% of 100 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


40% of 100 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


60% of 100 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3


80% of 100 Streams Off-target
ARL
25.00
20.00
15.00
10.00
5.00
0.00
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
% of Total Sampled
0.5
1
1.5
2
2.5
3

APPENDIX 3B

VISUAL BASIC PROGRAM FOR PROBABILITY OF DETECTION

The following computer code was written using Visual Basic for Applications and Microsoft® Excel 97.

```
Option Explicit
Dim K() As Single
Dim M() As Integer
Dim Mstar() As Integer
Dim Response, Msg, Style, Title, DefaultAnswer

'Integer Variables
Dim N As Integer
Dim P As Integer
Dim Q As Integer
Dim Bad As Integer
Dim Pointer As Integer
Dim Sample As Integer
Dim Need As Integer
Dim Coll As Integer
Dim LookHere As Integer
Dim ZZ As Integer

'Real Variables
Dim SumMK As Single
Dim ZStat1 As Double
Dim ZStat2 As Double
Dim Numerator As Double
Dim Denominator As Double
Dim SeqProb As Double
Dim SignalProb As Double
Dim TotalProb As Double

' Boolean Variables
Dim Finished As Boolean
Dim EnoughLeft As Boolean
Dim BackingUp As Boolean
Dim FirstTime As Boolean
Dim GoingAgain As Boolean

' Counter Variables
Dim I As Integer, J As Integer, Z As Integer, Count As Integer
```

```
Sub ProbDetect()
    IntroQuestion
    Do
        If Not Finished Then
            Initialize
            GetInputs
        End If
        Do Until Finished
            AllocateSamples
            If EnoughLeft Then RunIt
            ResetPointer
        Loop
        ShowOutput
        CopyResult
    Loop Until Not GoingAgain
    CleanUp
End Sub


Sub IntroQuestion()
    FirstTime = False

    Range("B1").Select
    If ActiveCell = Empty Then
        DefaultAnswer = vbDefaultButton1
        Msg = "Do you want to start from scratch?"
    Else
        DefaultAnswer = vbDefaultButton2
        Msg = "Do you want to overwrite existing values?"
    End If
    Style = vbYesNoCancel + vbQuestion + DefaultAnswer
    Title = "Probability of Detection"
    Response = MsgBox(Msg, Style, Title) ' Display message.
    If Response = vbYes Then
        FirstTime = True
        Finished = False
    ElseIf Response = vbNo Then
        FirstTime = False
        Finished = False
    ElseIf Response = vbCancel Then
        Finished = True
    End If
End Sub
```

```
Sub Initialize()
    Range("B1").Select
    If FirstTime Then
        Cells.Delete
        Cells.Font.Bold = False
        Cells.Font.ColorIndex = xlColorIndexAutomatic
        Cells.NumberFormat = "General"
        ActiveWorkbook.Names.Add Name:="Streams", RefersToR1C1:="=R1C2"
        ActiveWorkbook.Names.Add Name:="BadGroups",
            RefersToR1C1:="=R2C2"
        ActiveWorkbook.Names.Add Name:="SampleSize",
            RefersToR1C1:="=R3C2"
        ActiveWorkbook.Names.Add Name:="M0", RefersToR1C1:="=R3C5"
        ActiveWorkbook.Names.Add Name:="TotalProb",
            RefersToR1C1:="=R5C2"
    Else
        Columns("H:Z").Select
        Selection.Delete Shift:=xlToLeft
    End If
    Bad = 0: Pointer = 0
    Sample = 0: Need = 0
    SumMK = 0: ZStat1 = 0
    Numerator = 0: Denominator = 0
    SeqProb = 0: SignalProb = 0: TotalProb = 0
    Finished = False: EnoughLeft = True
    BackingUp = False: GoingAgain = True

    I = 0: J = 0: Z = 0: Count = 0
End Sub


Function GetInputs()
    If FirstTime Then
        Title = " "
        Msg = "How many streams in the process?"
        P = InputBox(Msg, Title, , 400, 1500)
        Range("Streams").Select
        With Selection
            .HorizontalAlignment = xlLeft
            .Value = P
            .Offset(, -1).ColumnWidth = 23
            .Offset(, -1).HorizontalAlignment = xlRight
            .Offset(, -1).Font.Bold = True
            .Offset(, -1).Value = "Streams(P) ="
        End With
    Else
        P = Range("Streams").Value
    End If
```

Continued …

```
Do
    If FirstTime Or Q > P Then
        Title = " "
        Msg = "Number of streams = " & P & Chr(13) & Chr(13) & "How
              many subsets of bad streams?"
        Q = InputBox(Msg, Title, , 400, 1500)
        Range("BadGroups").Select
        With Selection
            .HorizontalAlignment = xlLeft
             .Value = Q
            .Offset(, -1).HorizontalAlignment = xlRight
             .Offset(, -1).Font.Bold = True
              .Offset(, -1).Value = "# of Bad Groups(Q) ="
            .Offset(, 1).HorizontalAlignment = xlLeft
             .Offset(, 1).Font.Bold = True
              .Offset(, 1).Value = "'====>"
        End With
    Else
        Q = Range("BadGroups").Value
    End If
Loop While Q > P

ReDim M(0 To Q), Mstar(0 To Q), K(0 To Q)
Range("M0").Select
With Selection
    If FirstTime Then
        .Offset(-1, -1).HorizontalAlignment = xlCenter
         .Offset(-1, -1).Font.Bold = True
          .Offset(-1, -1).Value = "Group #"
        .Offset(-1).HorizontalAlignment = xlCenter
         .Offset(-1).Font.Bold = True
          .Offset(-1).ColumnWidth = 10
           .Offset(-1).Value = "Streams(M)"
        .Offset(-1, 1).HorizontalAlignment = xlCenter
         .Offset(-1, 1).Font.Bold = True
          .Offset(-1, 1).Value = "k"
    End If
    LookHere = Q + 1
    While LookHere > 0
        If .Offset(LookHere, -1).Value > 0 Then
            .Offset(LookHere, -1).Delete Shift:=xlUp
            .Offset(LookHere).Delete Shift:=xlUp
            .Offset(LookHere, 1).Delete Shift:=xlUp
        Else
            LookHere = -1
        End If
    Wend
```

Continued …

```
    For J = 1 To Q
        .Offset(J, -1).HorizontalAlignment = xlCenter
         .Offset(J, -1).Value = J
        If .Offset(J).Value = Empty Then
            Title = "Inputs For Bad Sub-Group #" & J
            Msg = Chr(13) & "                          How many bad
                    streams in subset #" & J
            M(J) = InputBox(Msg, Title)
            .Offset(J).HorizontalAlignment = xlCenter
             .Offset(J).Value = M(J)
            Title = "Inputs For Bad Sub-Group #" & J
            Msg = Chr(13) & "                          Number of bad
                    streams in subset " & J & " is " & M(J) &
                    Chr(13) & Chr(13) & "                        Size of
                    Shift for Subset #" & J
            K(J) = InputBox(Msg, Title)
            .Offset(J, 1).HorizontalAlignment = xlCenter
             .Offset(J, 1).Value = K(J)
        Else
            M(J) = .Offset(J).Value
            K(J) = .Offset(J, 1).Value
        End If
        Bad = Bad + M(J)
        If Bad > P Then
            .Offset(J).ClearContents
            Bad = Bad - M(J)
            J = J - 1
            Msg = "Choose fewer bad streams or Abort"
            Style = vbOKOnly + vbExclamation
            Title = "Too Many Bad Streams Selected."
    ' Display message.
            Response = MsgBox(Msg, Style, Title)
        End If
    Next J
    M(0) = P - Bad
    .HorizontalAlignment = xlCenter
     .Value = M(0)
    If FirstTime Then
        .Offset(, -1).HorizontalAlignment = xlCenter
         .Offset(, -1).Value = 0
        .Offset(, 1).HorizontalAlignment = xlCenter
        .Offset(, 1).Value = K(0)
    End If
End With
```

Continued …

```
    If FirstTime Then
    Title = "SAMPLE SIZE"
    Msg = "Number of streams = " & P & Chr(13) & "Number of subsets of
      bad streams = " & Q & Chr(13) & Chr(13) & "
      Number of Streams to Sample?"
    N = InputBox(Msg, Title, , 400, 1500)
        Range("SampleSize").Select
        With Selection
             .HorizontalAlignment = xlLeft
              .Value = N
             .Offset(, -1).HorizontalAlignment = xlRight
              .Offset(, -1).Font.Bold = True
               .Offset(, -1).Value = "Sample Size(N) = "
        End With
    Else
        N = Range("SampleSize").Value
    End If

    Coll = Range("M0").Offset(-1, 3).Column
    For J = 0 To Q + 4
        If J <= Q + 1 Then
            Columns(Coll + J).HorizontalAlignment = xlCenter
        Else
            Columns(Coll + J).Style = "Percent"
        End If
        If J = 0 Or J > (Q + 1) Then
            Columns(Coll + J).Font.Bold = True
        End If
        If J = (Q + 2) Then
            Columns(Coll + J).Font.ColorIndex = 41
        ElseIf J = (Q + 3) Then
            Columns(Coll + J).Font.ColorIndex = 50
        ElseIf J = (Q + 4) Then
            Columns(Coll + J).Font.ColorIndex = 3
            Columns(Coll + J).NumberFormat = "0.0%"
        End If
    Next J
    Range("M0").Select
    With Selection
          .Offset(-1, 3).Value = "Number"
        For J = 0 To Q
             .Offset(-1, 4 + J).Font.Bold = True
              .Offset(-1, 4 + J).Value = "Mstar " & J
        Next J
        .Offset(-1, 5 + Q).HorizontalAlignment = xlCenter
          .Offset(-1, 5 + Q).Value = "Seq"
        .Offset(-1, 6 + Q).HorizontalAlignment = xlCenter
          .Offset(-1, 6 + Q).Value = "Signal"
        .Offset(-1, 7 + Q).HorizontalAlignment = xlCenter
          .Offset(-1, 7 + Q).Value = "Total"
    End With
Denominator = Application.Combin(P, N)
End Function
```

```
Sub AllocateSamples()
    Need = N - Sample
    While (Need > 0) And EnoughLeft
        If (M(Pointer) - Mstar(Pointer)) >= Need Then
            Mstar(Pointer) = Mstar(Pointer) + Need
            Sample = Sample + Need
        Else
            Mstar(Pointer) = M(Pointer)
            Sample = Sample + Mstar(Pointer)
            If Pointer = Q Then
                EnoughLeft = False
            Else
                Pointer = Pointer + 1
            End If
        End If
        Need = N - Sample
    Wend
    Z = Pointer + 1
    While Z <= Q
        Mstar(Z) = 0
        Z = Z + 1
    Wend
End Sub


Sub RunIt():
    Numerator = 1
    SumMK = 0
    For J = 0 To Q
        Numerator = Numerator * Application.Combin(M(J), Mstar(J))
        SumMK = SumMK + (Mstar(J) * K(J))
    Next J
    SeqProb = Numerator / Denominator
    ZStat1 = 3 - (Sqr(N) / N) * (SumMK)
    ZStat2 = -3 - (Sqr(N) / N) * (SumMK)
    SignalProb = 1 - (Application.NormSDist(ZStat1) -
      Application.NormSDist(ZStat2))
    TotalProb = TotalProb + (SeqProb * SignalProb)
    Count = Count + 1
    Range("M0").Select
    With Selection
        .Offset(-1 + Count, 3).Value = Count
        For J = 0 To Q
            .Offset(-1 + Count, 4 + J).Value = Mstar(J)
        Next J
        .Offset(-1 + Count, 5 + Q).Value = SeqProb
        .Offset(-1 + Count, 6 + Q).Value = SignalProb
        .Offset(-1 + Count, 7 + Q).Value = SeqProb * SignalProb
    End With
End Sub
```

```
Sub ResetPointer()
    Mstar(Pointer) = Mstar(Pointer) - 1
    Sample = Sample - 1
    If Pointer < Q Then
        Pointer = Pointer + 1
    Else
        While Mstar(Pointer) > 0
            Mstar(Pointer) = Mstar(Pointer) - 1
            Sample = Sample - 1
        Wend
        BackingUp = True
        While BackingUp
            Pointer = Pointer - 1
            If Pointer < 0 Then
                Finished = True
                BackingUp = False
            ElseIf Mstar(Pointer) > 0 Then
                BackingUp = False
                Mstar(Pointer) = Mstar(Pointer) - 1
                Sample = Sample - 1
                Pointer = Pointer + 1
                EnoughLeft = True
            End If
        Wend
    End If
End Sub

Sub ShowOutput()
    '   MsgBox ("Overall Probability is " & TotalProb)
    Range("TotalProb").Select
    With Selection
        .Offset(, -1).HorizontalAlignment = xlRight
         .Offset(, -1).Font.Bold = True
          .Offset(, -1).Font.ColorIndex = 5
           .Offset(, -1).Value = "Probability of Detection ="
        .HorizontalAlignment = xlCenter
         .Style = "Percent"
          .NumberFormat = "0.00%"
           .Font.Bold = True
            .Font.ColorIndex = 5
             .Value = TotalProb
        .Offset(1, -1).HorizontalAlignment = xlRight
         .Offset(1, -1).Font.ColorIndex = 9
          .Offset(1, -1).Value = "in " & Count & " sequences"
        .Offset(2, -1).HorizontalAlignment = xlCenter
         .Offset(2, -1).Font.Bold = False
          .Offset(2, -1).Font.ColorIndex = 50
           .Offset(2, -1).Value = "Assoc ARL ="
        .Offset(2).HorizontalAlignment = xlCenter
         .Offset(2).Font.Bold = False
          .Offset(2).Font.ColorIndex = 50
           .Offset(2).Value = 1 / TotalProb
    End With
End Sub
```

```
Sub CopyResult()
'
' CopyResult Macro
' Macro recorded 5/13/98 by Jeffrey Wayne Lanning
'

'
    Selection.Copy
    Sheets(Results").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
      SkipBlanks:= _
        False, Transpose:=False
    If M(Q) < P Then
        ActiveCell.Offset(1).Select
        If M(Q) < 10 Then
            M(Q) = M(Q) + 1
        Else
            M(Q) = M(Q) + 5
        End If
        Finished = False
    Else
        If M(Q) = 5 Then
            ZZ = 10
        Else
            ZZ = M(Q)
        End If
        ActiveCell.Offset((((ZZ / 5) - 2) * 4) - M(Q), 2).Select
        GoingAgain = False
    End If
    Sheets("Sheet2").Select
    Range("M0").Offset(Q).Select
    Selection.Value = M(Q)
End Sub


Sub CleanUp()
    For J = 0 To Q
        M(J) = 0
        Mstar(J) = 0
        K(J) = 0
    Next J
End Sub
```

CHAPTER 4

ADAPTIVE MONITORING TECHNIQUES

Introduction

Earlier we introduced the simple, yet effective, Shewhart $\overline{X}$ chart. We also pointed out that while the Shewhart chart is popular for monitoring the mean of a process, it is often too slow to respond to situations characterized by small process shifts, or a drifting mean. To compensate for this, several enhancements and alternatives have been proposed to improve the Shewhart chart's performance. Alternative charts such as the cumulative sum, exponentially weighted moving average, and hybrid were reviewed earlier. Enhancements previously discussed included Shewhart charts with warning limits, and Shewhart charts with runs rules.

Each of the Shewhart chart alternatives and enhancements discussed to this point have assumed a constant interval between samples as well as constant sample size. An enhancement that warrants further consideration does not rely on such assumptions. A monitoring technique that makes use of the most recent information on the $\overline{X}$ chart and modifies the sample size or sample interval accordingly is called an adaptive monitoring chart. This adaptive approach essentially says: If the current sample's mean is within the chart limits, but remote from the target value, either don't wait a full time interval before forming the next sample, or let the next sample be larger than normal. On the other hand, if the mean of the current sample is relatively close to target, the next sample need not be

taken immediately, or the next sample might be smaller than normal. In this fashion the sampling rate or sample size adapts based on the most recent sample information.

The idea of employing an adaptive approach to statistical monitoring is an intuitive one. When an observed sample is near the process target value we are confident that the process is behaving as desired and are content to let the process run for a while. However, when a point plots near the chart limits, we begin to wonder whether the process mean may have moved off-target and tend to watch things more closely.

While an on-target process may produce a sample near a chart limit, this is a relatively rare event or else the limits would not be where they are. On the other hand, if the process mean has shifted to a higher value, a realized sample close to the upper limit is much more reasonable. So, if a sample plots near a limit, it makes intuitive sense to watch the process more closely. This may be accomplished by re-sampling immediately, or taking a more thorough sample next time.

As a simple example of this method's appeal, let's consider an average car owner. Most drivers have a general idea of how many days, or how many miles they can drive on a tank of gas, even if they don't routinely monitor their miles per gallon (mpg). As long as trips to the gas station occur after the usual amount of time or distance the mpg is not tracked very closely, and maybe only figured exactly while on long trips. However, if suddenly the gas tank needs to be filled more frequently than normal, the driver is quick to determine his mpg to the exact digit after every fill-up. This is an example of adaptive monitoring. While things are on-target less items are monitored less frequently, but when the process is relatively far from target the situation is monitored more closely in an

attempt to determine if the process has changed or is simply experiencing natural variation.

When constructing a typical Shewhart $\overline{X}$ chart, appropriate consideration must be given to selection of a rational subgroup, or sample size, as well as a practical sampling frequency. In many manufacturing situations, physical constraints may dictate upper and lower boundaries for possible sample sizes and frequencies. Generally these decisions are made after considering the size of shift in the process that it is necessary to detect (Montgomery (1996)). While samples taken more frequently and of larger size provide better protection against process shifts than few samples of small size, large, frequent samples are often not feasible due to limited sampling resources. Traditionally a sampling plan is developed which provides adequate protection against expected process shifts. However, it may be feasible to take larger, or more frequent samples occasionally if the remaining samples can be made respectively smaller, or less frequent. This is the notion behind adaptive process monitoring.

In the following section the development of the adaptive methods will be laid out followed by an examination of the methodology for each of three adaptive approaches. We will see that by varying the sample size or sample interval, we can improve upon the results obtained with standard, fixed-interval, fixed-sample size $\overline{X}$ charts. We will also see that a combined approach, allowing the interval and sample size to be adaptive, provides the best monitoring results. Following the discussion of how each modification works, we will consider how adaptive techniques can be used in a multiple stream situation.

Background

Adaptive process monitoring is comprised of two primary approaches, *variable sample interval* (VSI), and *variable sample size* (VSS). A third technique called *variable sample size and interval* (VSSI) is, as its name suggests, a combination of the two previous methods. Before we consider the background for each of these techniques let's develop definitions and notation for a representative process to be monitored.

Assume we have a process with a quality characteristic X to be monitored. Let X be normally distributed with a target mean μ_0 and a standard deviation σ which is both known and constant. If $\overline{X}_i$ is the mean of the i^{th} sample with an associated sample size of n, then standardizing we obtain

$$Z_i = \frac{\overline{X}_i - \mu_0}{\sigma/\sqrt{n}} \tag{4-1}$$

where $Z_i \sim N(0,1)$. The use of Z_i will be especially important in constructing variable sample size charts. Since the computation of upper and lower chart limits as well as threshold values involve the sample sizes, if we allow variable sample sizes we will also have variable limits. By standardizing the values first, we can monitor the process using a chart with only one set of limits and threshold values.

We will consider the process on-target if the process mean, μ, equals the target mean, μ_0. If any point plots beyond the chart limits the chart will be considered to have signaled giving an indication that the process has shifted to a new mean, say μ_1, and the process will be stopped to allow for correction of the associated assignable cause.

Shifts in the process mean will be expressed in terms of process standard deviation units using δ. For example, a shift of $\delta_{1.5}$ would indicate an upward shift of the process mean by 1.5 standard deviations. We will also assume the use of a standardized $\overline{X}$ chart. Recall that such a chart has a center line at zero and chart limits at $\pm 3\ \sigma$. Finally, we will define the sample size and sampling interval of the fixed-interval, fixed-sample size chart to be n_0 and t_0 respectively.

Variable Sample Interval. The VSI approach to adaptive monitoring makes improvements on traditional $\overline{X}$ charts by allowing the frequency of samples taken to vary. The technique uses information contained in the previous sample to determine when the next sample should be taken. While the process is relatively close to target, samples are taken infrequently and the frequency is increased as the chart limits are approached.

The idea of modifying the waiting times in Shewhart $\overline{X}$ charts is introduced by Reynolds, Amin, Arnold, and Nachlas (1988) and extended through independent work by Runger and Pignatiello (1991). Runger and Pignatiello also include information on practical implications of VSI monitoring. Other independent work includes Cui and Reynolds (1988) who consider VSI monitoring using $\overline{X}$ charts enhanced with runs rules and Chengular, Arnold, and Reynolds (1989) who introduce VSI techniques for multiparameter $\overline{X}$ charts. Concurrent work by Reynolds (1989), and Reynolds and Arnold (1989) consider optimal adaptive sampling schemes and obtain theoretical results similar to Runger and Pignatiello.

Runger and Pignatiello (1991) extend the VSI work of Reynolds, Amin, Arnold, and Nachlas by providing detailed performance calculations for many one-sided and two-sided adaptive Shewhart schemes. They point out that for the one-sided case the adaptive schemes provide the greatest improvement. Furthermore, they provide formulas for the performance of the adaptive charts in terms of proposed waiting times and analyze the effects of different choices. Finally, the two-sided dual waiting time chart is described and evaluated using formulas and tables.

Variable Sample Size. The VSS approach to adaptive monitoring makes improvements on traditional $\overline{X}$ charts by allowing the size of samples taken to vary. The technique uses information contained in the previous sample to determine how large the next sample taken should be. While the process is relatively close to target, samples are relatively small in size and the sample size is increased as the chart limits are approached.

The idea of modifying the sample size in Shewhart $\overline{X}$ charts probably grew out of double-sampling acceptance plans. Typical acceptance sampling plans are methods of making decisions regarding the appropriate disposition of inspected material by taking a sample from the material in question (Montgomery (1996)). The decision process is sometimes called *sentencing*. A double-sampling acceptance plan is a procedure where one of the possible results of the sentencing from an initial sample is to take a second sample. Montgomery indicates the primary advantage of double-sampling plans as compared with single-sample plans is a reduction in the number of total inspections

required. Acceptance sampling is discussed in detail in several texts including Montgomery (1996) and Duncan (1986).

The concept of varying the sample size within an $\overline{X}$ charting scheme was introduced by Flaig (1991). He divided the region between the chart limits and the center-line into three zones on each side of the center line. Depending on which zone contained the previous sample mean, a corresponding sample size was indicated for the next sample. The link to double-sampling acceptance plans is clearly seen in work by Daudin (1992). Rather than determining what the next regularly scheduled sample size should be, Daudin suggests collecting an additional, larger sample immediately and combining this information with the first sample. He provides a table of optimal double-sampling $\overline{X}$ charts for use with various associated fixed sample size Shewhart charts.

The concept of adapting the next sample to be taken using a dual-sample size scheme was proposed by Prabhu, Runger, and Keats (1993) and also studied by Costa (1994). These works closely parallel the VSI results mentioned above and will be the basis of the discussion to follow. Other related work of note addresses situations where, rather than being adaptive, the sample sizes are simply not uniform. That is, situations where the data, as collected, happens to have samples of various sizes due to circumstances such as lost data, etc. Burr (1969) weights each sample according to its size to estimate the standard deviation while Nelson (1990) considers standard deviation estimation when sample sizes are not uniform.

Variable Sample Size and Interval. The VSSI approach to adaptive monitoring makes improvements on traditional $\overline{X}$ charts by allowing the both the frequency and size of samples taken to vary. The technique uses information contained in the previous sample to determine when the next sample should be taken and how large the sample should be. While the process is relatively close to target, samples are small and taken infrequently with both the size and frequency being increased as the chart limits are approached.

The idea of combining the VSI and VSS monitoring approaches was first suggested by Prabhu, Montgomery, and Runger (1994). Their work is a natural progression of the research cited above. Rendtel (1990) propose a similar adaptive approach for CUSUM *p*-charts, allowing both sample size and interval to vary.

Prabhu, Montgomery, and Runger (1994) provide detailed performance calculations for their combined adaptive scheme. Their results show that the combined approach is better than the pure VSS approach in terms of the average time to detect an off-target process. The combined approach is also seen to outperform the pure VSI approach for small shifts in the process mean, although the VSI approach seems to have a slight advantage for large shifts of the process mean. The advantage of the VSI in the latter case, however, is very slight and most likely attributable to the more frequent large sample sizes required of the VSSI technique as the process shifts further from the target.

The following sections will discuss these adaptive monitoring schemes and provide an example for each method. This discussion will serve as a foundation for developing an adaptive approach to fractional monitoring of multiple stream processes.

A more detailed, general discussion of these and several other adaptive approaches in quality monitoring is given by Tagaras (1998).

Variable Sample Interval Methodology

Before setting up a VSI monitoring scheme, minimum and maximum allowable intervals between samples need to be established. Minimum times are often driven by physical or practical factors such as time to acquire the sample, time to measure the items, and time necessary to record the results and prepare for the next sample. Maximum interval times are generally based on comfort level – how long one is willing to let the process go without at least a small sample being taken.

While we might envision a myriad of interval times being used for each of several warning zones between the target value and chart limits, Reynolds (1989) and Reynolds and Arnold (1989) showed that only two intervals are needed for any process which can be modeled as a Markov chain. They also showed that the technique works best when the two values are spread far apart. This result allows us to make use of the minimum practical and maximum allowable intervals. Both Reynolds *et al* (1989) and Runger and Pignatiello (1991) make use of this dual waiting time approach in constructing VSI monitoring schemes. Runger and Pignatiello also impose a constraint based on a symmetry requirement for the two-sided monitoring problem that simplifies the theory and is easily understood in practice.

Since the VSI adaptive approach constructs charts where the interval between samples is allowed to vary, the traditional method of comparing different monitoring

schemes using the average run length (ARL) is not appropriate. Since the ARL measures the number of samples before a chart signals, this approach is only valid if each scheme uses a fixed sampling interval. Runger and Pignatiello (1991) point out that since the VSI approach intentionally varies the timing between samples, the average number of samples taken before detection is no longer an appropriate measure. They suggest the use of the mean time until detection. By careful construction of the VSI sampling plan they cause the average time between samples of an on-target variable interval scheme to be equal to a comparable on-target fixed rate interval scheme. So in the VSI case, the average time between samples equals the exact time between samples of the fixed rate case thereby allowing for direct comparisons of on-target ARLs.

To construct a VSI $\overline{X}$ chart using the approach of Runger and Pignatiello we need to identify sampling intervals both less than and greater than the fixed interval chart, t_0. Let t_1 be the shorter interval and t_2 be the longer interval. If we also allow w_1 to be the threshold value for switching between sampling intervals we can define an adaptive sampling interval function for the current sample (i), based on the value of the previous sample, Z_{i-1}.

$$t(i) = \begin{cases} t_1 \text{ if } w_1 < Z_{i-1} < UCL \\ t_2 \text{ if } -w_1 \le Z_{i-1} \le w_1 \\ t_1 \text{ if } LCL < Z_{i-1} < -w_1 \end{cases} \qquad (4\text{-}2)$$

Equation 4-2 shows that if the previous sample (Z_{i-1}) falls beyond the threshold value (w_1), but remains within the chart limits (UCL and LCL) we will use the short sampling interval, t_1 (greater frequency) for the current sample (i). However, if the

previous sample is located within the threshold values, the longer sample interval, t_2, will be used. Figure 4-1 shows the function in a graphical format adapted from Runger and Pignatiello (1991).

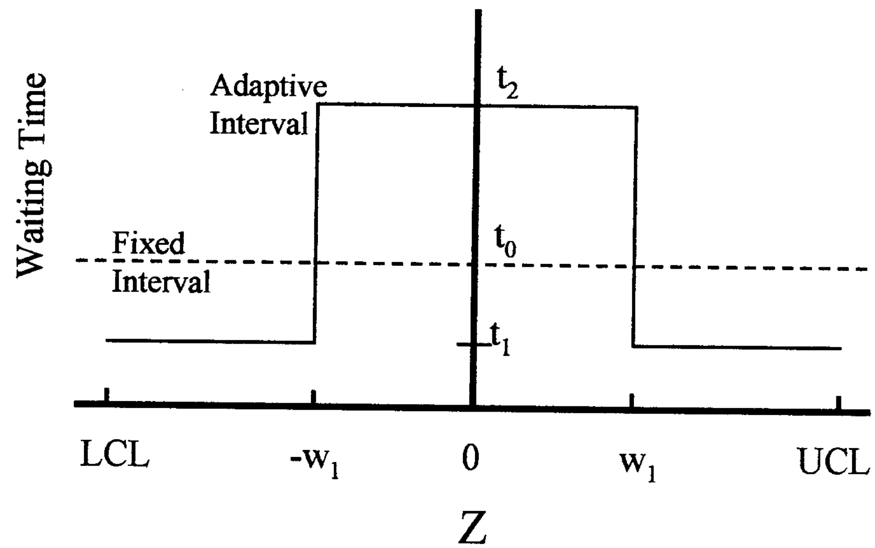


FIGURE 4-1. Fixed Interval and Adaptive Interval Waiting Time Functions

A note of practical consideration is worth mentioning here. While any manageable values for t_1 and t_2 can be selected, we might want to consider the potential consequences of various choices. For example, if currently sampling every hour, but an adaptive approach is desired, we might decide to sample as often as every 15 minutes and allow the process to run without being sampled for up to 4 hours. If the average time between samples is to remain equal to 1 hour, the threshold value will be such that we expect to be within the threshold limits only about 20 percent of the time. The practical implications are that samples will need to be taken at 15 minute intervals much more

often than not. So, if taking samples every 15 minutes is possible, but constitutes a major inconvenience, different interval values ought to be chosen.

As implied in the previous discussion, the selection of upper and lower allowable sampling intervals will determine the value of the threshold value. Runger and Pignatiello give an equation similar to the following for determining the proper value for w_1 in a two-sided monitoring situation.

$$w_1 = \Phi^{-1}\left[\frac{\Phi(UCL)(2)(t_0 - t_1) + (t_2 - t_0)}{2(t_2 - t_1)}\right] \tag{4-3}$$

where Φ is the cumulative standard normal function. Note that if t_0 is equal to 1 time unit, say 1 hour, then equation 4-3 simplifies to

$$w_1 = \Phi^{-1}\left[\frac{\Phi(UCL)[2 - 2t_1] + t_2 - 1}{2t_2 - 2t_1}\right] \tag{4-4}$$

which is the result given by Runger and Pignatiello.

As an example of this procedure, consider a simple filling operation where plastic bottles are filled one at a time with a liquid to a target value, $\mu_0 = 2000$ ml. The standard deviation of the process is stable and known to be equal to $\sigma = 6$ ml. This process is susceptible to shifts in the process mean resulting in either over or under-filled bottles. Under-filled bottles are a problem as the bottle no longer contains the advertised amount of product while over-filling reduces product yield and overall profits. Since replacing or adjusting the filling valve requires shutting down the filling machine, unnecessary valve adjustments are to be avoided.

Rational subgroups from this process consist of 5 bottle samples. Currently a sample is taken once every hour and, although the sample interval could be allowed to vary, the average inter-sample time is desired to remain at one hour. The upper and lower chart limits for this process are $UCL_{\overline{X}} = \mu_0 + 3\sigma/\sqrt{n} = 2000 + 3 \cdot 6/\sqrt{5} \cong 2008$ and $LCL_{\overline{X}} \cong 1992$. Due to constraints associated with the time required to pull a sample, measure each bottle, record the values, and return them to the bottling line; the minimum time between samples is estimated at 15 minutes. Management does not feel comfortable letting the line operate for more than 2 hours between samples. Therefore we shall set $t_1 = 0.25$ and $t_2 = 2.0$.

Using this information and equation 4-4 we can find an appropriate value for w_1.

$$w_1 = \Phi^{-1}\left[\frac{\Phi(3)(2)(1-.25)+(2-1)}{2(2-.25)}\right] = 0.56$$

We can now establish upper and lower sampling thresholds of $\mu_0 + w_1\,\sigma/\sqrt{n} = 2001.5$ and 1998.5 respectively. Figure 4-2 shows the chart with threshold limits and how samples in each zone will affect the next sample taken.

Now assume the first sample from the bottling operation occurs at 8 o'clock a.m., $t(1) = $ 08:00 and yields $\overline{X}_1 = 1995.6$. As this value is within the chart limits, but outside the threshold limits, we see from Figure 4-2 that the next sample should be taken at $t(2) = t(1) + t_1 =$ 08:15. If the average of the next sample, $\overline{X}_2$ (that is the sample taken at 08:15), falls within the threshold limits, say 2001.2, we would schedule the next sample at $t(3) = t(2) +$ 2:00 = 10:15.

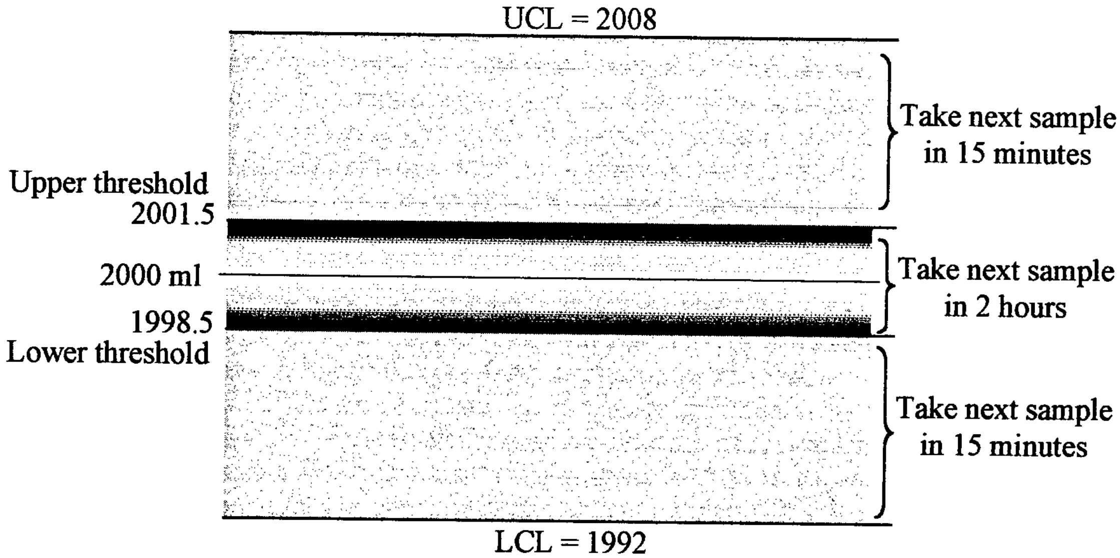


FIGURE 4-2. VSI Chart Showing Threshold and Limit Values and Appropriate Actions

This procedure continues until a sample mean exceeds the chart limits. The tables given by Runger and Pignatiello indicate that if the process mean should shift by, say 2.0σ to a new mean of $2000 + 2(6) = 2012$ the chart will signal within 2.6 hours on average as compared with 5.8 hours using a standard, fixed-interval Shewhart chart. This will result in detecting shifts of this size roughly twice as soon with the VSI adaptive approach as we would with the standard, fixed-interval scheme.

Being able to detect off-target processes quickly is often the goal of switching to a new monitoring scheme, however, this adaptive technique may be used to enhance process performance in other ways. For example, if the current process seems to be identifying off-target conditions at a rate that is deemed acceptable, an adaptive approach could be used to maintain that level of protection and free up more of an operator's time.

This in turn might allow for a reduction in staffing, or enable the operation to expand without a need to greatly increase personnel. The point is that we often look to make improvements in the ability to monitor a process by detecting off-target situations more quickly when in-fact more significant improvements might be possible using a new approach to maintain a current level of risk, but free up other process resources.

Variable Sample Size Methodology

Much like the VSI case, the VSS monitoring scheme requires that minimum and maximum allowable *sample sizes* be initially established. Maximum sample sizes are often driven by physical or practical factors such as available space, available sample collection time, and ability of machinery and/or operators to collect large sample sizes. Minimum sample sizes will often be a single item produced, but may also be driven by how few samples one is willing to measure and still continue normal process operation, or what constitutes a logical definition of a minimum sub-group.

We might envision several threshold levels between the target value and chart limits each with a corresponding sample size. Indeed Flaig (1991) proposed two sets of threshold values on either side of the center-line with three different sample sizes. Prabhu, Runger, and Keats (1993) showed that simply using two sample sizes yields excellent results. This result allows for simple implementation of the procedure and allows us to make use of the minimum practical and maximum allowable sample sizes. Both Prabhu, Runger, and Keats (1993) and Costa (1994) use this approach in

constructing VSS monitoring schemes, and we will take advantage of the technique in this section.

Unlike the VSI adaptive approach where the traditional method of comparing different monitoring schemes using the ARL was not appropriate, the VSS approach could be compared to fixed-sample size Shewhart techniques using standard definitions of ARL. Since the ARL measures the number of samples taken before a chart signals, and the VSS scheme uses a fixed sampling interval, the definition technically holds. However, Prabhu, Runger, and Keats (1993) use careful construction of the VSS plan to force the average sample size taken to be equal to a comparable on-target fixed sample size scheme to avoid misleading results obtained by taking the same number of samples but of varying sample sizes. So in the VSS case, the average sample size equals the exact sample size of the fixed chart case thereby removing any discrepancies in comparisons of on-target ARLs.

To construct a VSS $\overline{X}$ chart using the approach of Prabhu, Runger, and Keats (1993) we will need to identify sample sizes both less than, and greater than the sample size on the fixed chart, n_0. Let n_1 be the smaller sample size and n_2 be the larger sample size. If we also allow w_2 to be the threshold value for switching between sample sizes, we can define an adaptive sample size function for the current sample (i), based on the value of the previous sample, Z_{i-1}.

$$n(i) = \begin{cases} n_2 \text{ if } w_2 < Z_{i-1} < UCL \\ n_1 \text{ if } -w_2 \le Z_{i-1} \le w_2 \\ n_2 \text{ if } LCL < Z_{i-1} < -w_2 \end{cases} \tag{4-5}$$

Equation 4-5 shows that if the previous sample (Z_{i-1}) falls beyond the threshold values ($-w_2$, or w_2), but remains within the chart limits (*UCL* and *LCL*) we will use the large sample size, n_2 for the current sample (i). However, if the previous sample is located within the threshold values, the smaller sample size, n_1, will be used. Figure 4-3 shows the function in a graphical format similar to that used in the previous section.

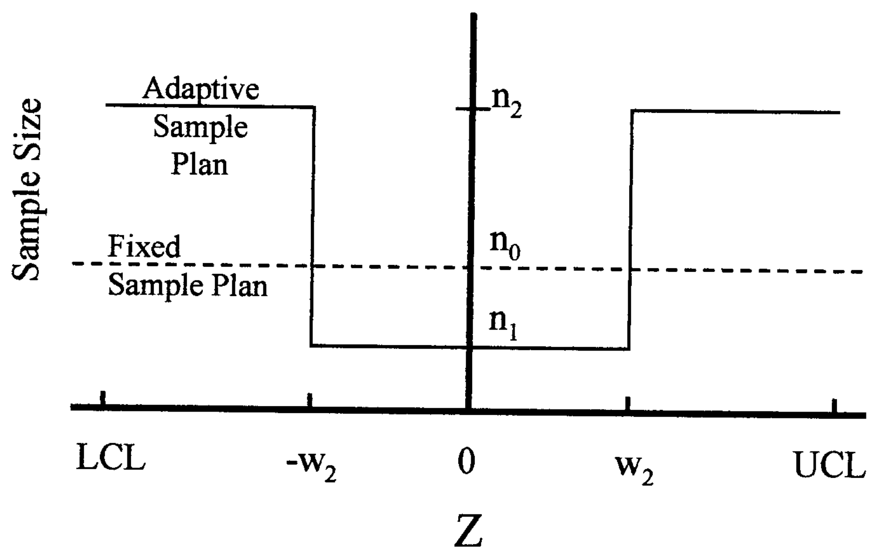


FIGURE 4-3. Fixed Interval and Adaptive Interval Waiting Time Functions

Analogous to the caution mentioned in the previous section, we need to make informed choices when choosing values for n_1 and n_2. For example, if we are currently using a sample size of 5, but want to implement an adaptive approach we might decide

we could accept samples of as few as only 1 item and as many as 20. If we want the average sample size to remain equal to 5, the threshold value will be such that we expect to be within the threshold limits nearly 80 percent of the time. The practical implications are that we will need to take samples of only 1 item much more often than we will take samples of 20. So, if taking samples this small is acceptable only if it is a rare event, different sample size values ought to be chosen.

As implied in the previous discussion, the selection of upper and lower allowable sample sizes will determine the threshold value. Prabhu, Runger, and Keats (1993) give an equation similar to the following for determining the proper value for w_2 in a two-sided monitoring situation.

$$w_2 = \Phi^{-1}\left[\frac{\Phi(UCL)(2)(n_0 - n_2) + (n_1 - n_0)}{2(n_1 - n_2)}\right] \tag{4-6}$$

where Φ is the cumulative standard normal function.

As an example of this procedure, consider again the simple filling operation where plastic bottles are filled with a liquid to a target value, $\mu_0 = 2000$ ml., with a stable and known process standard deviation of $\sigma = 6$ ml.

Rational subgroups from this process are taken once every hour. Currently each sample contains 5 bottles and, although the sample size requirement is flexible, the average sample size should equal 5 bottles with an inter-sample time of one hour. The upper and lower chart limits for this process are

$UCL_{\bar{X}} = \mu_0 + 3\sigma/\sqrt{n} = 2000 + 3 \cdot 6/\sqrt{5} \cong 2008$ and $LCL_{\bar{X}} \cong 1992$. Due to space

limitations and sample collection workload, the maximum sample size is limited to 20 bottles. Management has agreed to allow sample sizes as small as 2 bottles to be taken as long as the interval between samples does not exceed 1 hour (the current operating condition). Therefore we shall set $n_1 = 2$ and $n_2 = 20$.

Using this information and equation 4-6 we can find an appropriate value for w_2.

$$w_2 = \Phi^{-1}\left[\frac{\Phi(3)(2)(5-20)+(2-5)}{2(2-20)}\right] = 1.38$$

We can now establish upper and lower sampling thresholds of $\pm\, w$ on the standardized chart using Equation 4-1. For a non-standardized chart we obtain two sets of threshold values and chart limits. For n_1, the upper and lower threshold values are $\mu_0 + w_2\, \sigma/\sqrt{n_1} = 2005.85$ and 1994.15 respectively with control limits at 2000 ± 12.73. For n_2, the upper and lower threshold values are $\mu_0 + w_2\, \sigma/\sqrt{n_2} = 2001.85$ and 1998.15 respectively with control limits at 2000 ± 4.02.

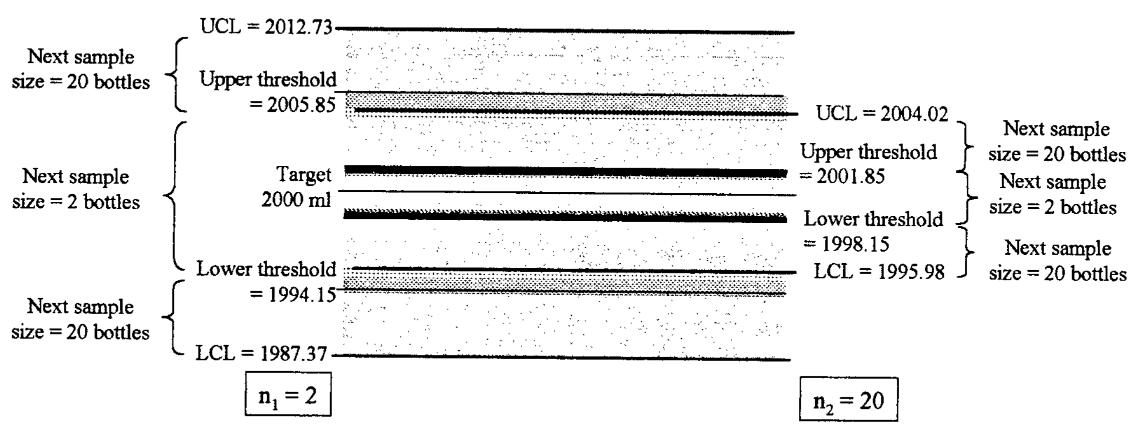


FIGURE 4-4. VSS Chart Showing Threshold and Limit Values and Appropriate Actions

Figure 4-4 shows the non-standardized chart with threshold limits. Compare this chart with the standardized chart shown in Figure 4-5. The standardized chart has fewer horizontal lines and generates less potential for confusion. Both charts show how samples in each zone will affect the next sample taken.

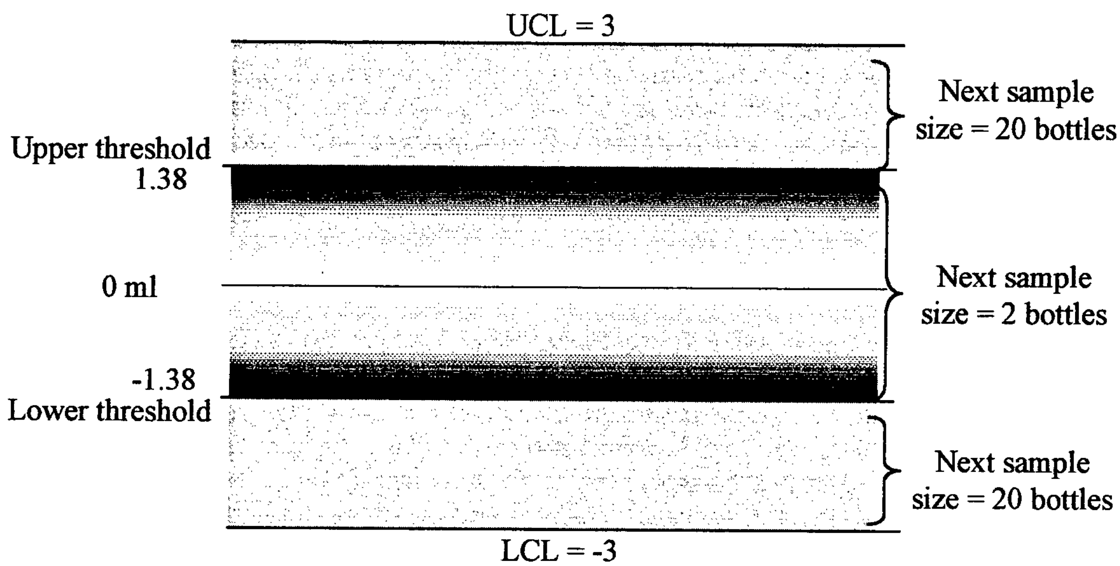


FIGURE 4-5. Standardized VSS Chart Showing Threshold and Limit Values and Appropriate Actions

Now assume the first sample from the bottling operation occurs at 8 o'clock a.m., $t(1) = 08{:}00$, is a sample of 20 bottles, and yields $\overline{X}_1 = 1995.6$. As this value is within the chart limits, but outside the threshold limits, we see from Figure 4-4 that the next sample size should also be 20 bottles, taken at $t(2) = t(1) + t_0 = 08{:}00 + 1{:}00 = 09{:}00$. If the average of the next sample, $\overline{X}_2$ (that is the sample taken at 09:00), falls within the

threshold limits, say 2002.2, we would sample only 2 bottles at the next scheduled sample time $t(3) = 10{:}00$.

This procedure continues until a sample mean exceeds the chart limits. The tables given by Prabhu, Runger and Keats indicate that if the process mean should shift by, say 1.0 σ to a new mean of 2000 + 2(3) = 2006 the chart will signal within 2.59 hours on average as compared with 4.50 hours using a standard, fixed-sample size Shewhart chart. This will result in detecting shifts of this size roughly twice as soon with the VSS adaptive approach as we would with the standard, fixed-sample size scheme. Unlike the VSI technique, the VSS approach to adaptive monitoring does reach a point of diminishing returns as shown in Table 4-1. Table 4-1 is a condensed version of Table 4 in Prabhu, Runger, and Keats (1993).

TABLE 4-1. Comparison of ARLs for VSS vs. Shewhart Schemes ($n_0 = 5$)
Condensed from Prabhu, et al. (1993)

	n_1	n_2	w	$\delta = 0$	$\delta = 0.5$	$\delta = 1.0$	$\delta = 2.0$
Shewhart chart				370.42	33.41	4.50	1.08
VSS chart							
	1	8	0.56	370.42	22.61	6.28	1.39
	1	12	0.91	370.42	15.34	2.57	1.62
	1	20	1.25	370.42	9.88	2.93	1.86
	2	8	0.67	370.42	23.06	2.90	1.29
	2	12	1.03	370.42	15.93	2.47	1.41
	2	20	1.38	370.42	10.29	2.59	1.51
	3	8	0.84	370.42	23.75	2.92	1.20
	3	12	1.22	370.42	17.01	2.44	1.25
	3	20	1.55	370.42	11.30	2.45	1.29
	4	8	1.15	370.42	25.14	2.98	1.12
	4	12	1.52	370.42	19.42	2.49	1.14
	4	20	1.85	370.42	13.97	2.46	1.15

Table 4-1 shows that for large shifts in the process mean, the fixed-sample size Shewhart chart outperforms the VSS chart. This result is most pronounced for VSS schemes utilizing large values for n_2. It is these large upper sample sizes which cause the poor performance. As the process moves further from target, we see from Figure 4-5 that we will be taking a greater percentage of large samples thereby causing the average number of samples to greatly increase and resulting in a slight inflation of the associated ARL.

As with the VSI approach, the VSS adaptive technique may be used to enhance process performance in ways other than improving time to detection. Rather than reducing employee workload by lengthening the average interval between samples, the VSS approach can allow for a reduction in the average number of samples collected. This might reduce the number of destructive tests required, free up more resources for other sampling procedures, or extend the life expectancy of the equipment involved in sample taking operations. Once again, the point is that we may be able to make significant improvements to overall process operation by maintaining a current level of risk protection, but freeing up other process resources along the way.

Variable Sample Size and Interval Methodology

In a process that combines the features of both the VSI and VSS schemes, the VSSI technique requires defining minimum and maximum values for both the intervals between samples and sample sizes. As before, minimum interval times and maximum

sample size values will likely be driven by physical or practical limitations while maximum times and minimum sample values will tend to be more arbitrary. The choice of threshold values will be slightly different. Generally the two sample sizes will be fixed along with the lower sampling interval rate and we will solve for the upper sampling interval rate.

While we might envision separate threshold values for determining when to switch sample size and when to adjust the sampling interval, Prabhu, Montgomery, and Runger (1994) show that selecting a single threshold value to indicate both changes is both simpler and sufficient.

Since the VSSI adaptive approach allows the interval between samples to vary, the traditional method of comparing different monitoring schemes using ARLs will, once again, not be appropriate. Like the VSI approach discussed earlier, Prabhu, Montgomery, and Runger use the mean time until detection, here called average time to signal (ATS). By careful construction of the VSSI sampling plan, they cause the average time between samples of an on-target combined adaptive scheme to be equal to a comparable on-target fixed rate interval scheme. Likewise the average sample size of the on-target adaptive scheme will equate to the on-target fixed sample size approach. So in the VSSI case, the average time between samples equals the exact time between samples of the fixed rate case and the average sample size equals the exact sample size of the fixed chart case, thereby allowing direct comparisons of on-target ARLs.

To construct a VSSI $\overline{X}$ chart using the approach of Prabhu, Montgomery, and Runger we will need to identify adaptive sampling intervals, t_1 and t_2 as well as adaptive sample sizes, n_1, and n_2. As before, we will let t_1 indicate the short interval, t_2 the long interval, n_1 the small sample size, and n_2 the large sample size. We will set $w = w_1 = w_2$ to indicate the threshold value for switching between both sampling intervals and sample sizes. The combined adaptive sampling function for the current sample, (i) based on the value of the previous sample, Z_{i-1} is given in Equation 4-7.

$$(t(i), n(i)) = \begin{cases} (t_1, n_2) \text{if } w < Z_{i-1} < UCL \\ (t_2, n_1) \text{if } -w \leq Z_{i-1} \leq w \\ (t_1, n_2) \text{if } LCL < Z_{i-1} < -w \end{cases} \tag{4-7}$$

Equation 4-7 shows that if the previous sample (Z_{i-1}) falls beyond the threshold value (w), but remains within the chart limits (UCL and LCL) we will use the short sampling interval, t_1 (greater frequency), and large sample size, n_2, for the current sample (i). However, if the previous sample is located within the threshold values, the longer sample interval, t_2, and small sample size, n_1, will be used. Figure 4-6 shows the function in a graphical format similar to those in the previous sections.

The practical considerations mentioned in the previous sections regarding making informed decisions about minimum sampling intervals and maximum sample sizes take on added urgency in the VSSI case. In the VSSI adaptive monitoring scheme we will be sampling more frequently when we find sample means plotting beyond the threshold values, and these samples will be large. For example, we may have decided a VSI

approach in which samples were allowed to adapt between 15 minutes and 4 hours worked fine for a fixed sample size of 5 units per sample. However, the VSSI technique will adapt both sampling interval and sample size so if we leave the VSI values the same, we may find ourselves sampling every 15 minutes with sample sizes of, say, 20 units per sample. Taking such large samples every 15 minutes may not be possible in which case different interval values and/or sample sizes should be chosen.

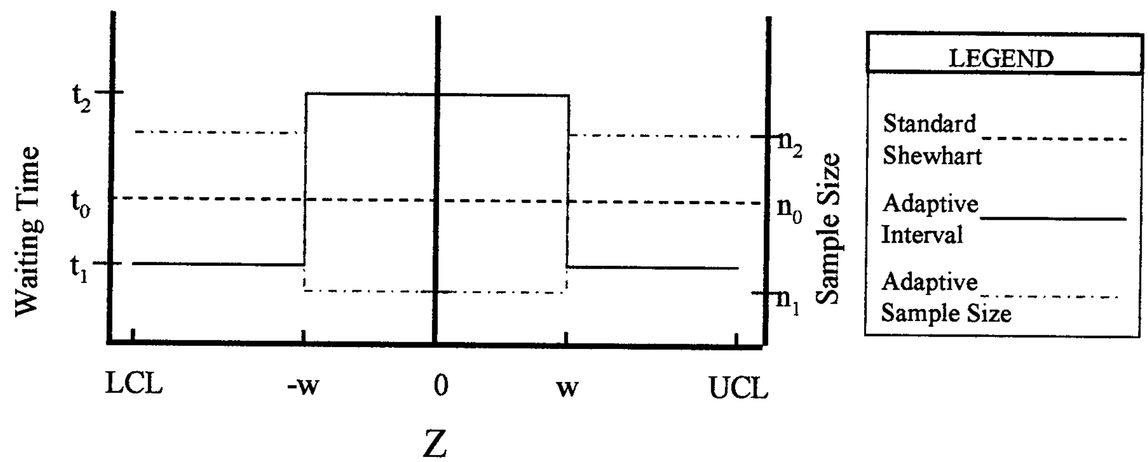


FIGURE 4-6. Fixed Interval and Adaptive Interval Waiting Time Functions

Unlike the VSI and VSS adaptive approaches where we had three parameters (VSI: t_1, t_2, and w_1, or VSS: n_1, n_2, and w_2), the VSSI scheme has five parameters (t_1, t_2, n_1, n_2, and w). If the average sampling interval of the VSSI adaptive approach is to equal the fixed sample interval of the standard Shewhart chart, and likewise for the sample size, two constraints are effectively generated. Prabhu, Montgomery, and Runger use these constraints to generate threshold equations for a two-sided monitoring situation. The

threshold equations below are the same as those seen in earlier sections (Equations 4-3, and 4-6 respectively).

$$w = \Phi^{-1}\left[\frac{\Phi(UCL)(2)(t_0 - t_1) + (t_2 - t_0)}{2(t_2 - t_1)}\right] \tag{4-8}$$

$$w = \Phi^{-1}\left[\frac{\Phi(UCL)(2)(n_0 - n_2) + (n_1 - n_0)}{2(n_1 - n_2)}\right] \tag{4-9}$$

We see from Equations 4-8 and 4-9 that selecting either upper and lower sampling intervals, or upper and lower sample sizes will allow us to solve for the threshold value, w. Now only one of the remaining parameter values needs to be specified in order to uniquely determine the other. Prabhu, Montgomery, and Runger suggest selecting desired sample sizes first in order to avoid any potential round off errors as sample sizes will necessarily be integer values. Having determined the threshold value, w, they further recommend fixing t_1 and solving for t_2 as the minimum sample interval is usually the least flexible of the two. Having fixed the parameters n_1, n_2, and t_1 we can solve for t_2 by setting equations 4-8 and 4-9 equal to one another.

$$\Phi^{-1}\left[\frac{\Phi(UCL)(2)(t_0 - t_1) + (t_2 - t_0)}{2(t_2 - t_1)}\right] = \Phi^{-1}\left[\frac{\Phi(UCL)(2)(n_0 - n_2) + (n_1 - n_0)}{2(n_1 - n_2)}\right]$$

$$\Phi(UCL)(2)(t_0 - t_1)(n_1 - n_2) + (t_2 - t_0)(n_1 - n_2) = \Phi(UCL)(2)(n_0 - n_2)(t_2 - t_1) + (n_1 - n_0)(t_2 - t_1)$$

$$t_2(n_1 - n_2) - t_2(n_1 - n_0) - t_2\Phi(UCL)(2)(n_0 - n_2)$$

$$= t_0(n_1 - n_2) - t_1(n_1 - n_0) - t_1\Phi(UCL)(2)(n_0 - n_2) - \Phi(UCL)(2)(t_0 - t_1)(n_1 - n_2)$$

$$t_2 = \frac{t_0(n_1 - n_2) - t_1[(n_1 - n_0) + \Phi(UCL)(2)(n_0 - n_2)] - \Phi(UCL)(2)(t_0 - t_1)(n_1 - n_2)}{[(n_1 - n_2) - (n_1 - n_0) - \Phi(UCL)(2)(n_0 - n_2)]}$$

Letting c equal the constant $\Phi(UCL)(2) = 1.9973$ we obtain a final equation similar (and equivalent) to that given by Prabhu, Montgomery, and Runger

$$t_2 = \frac{t_0(n_1 - n_2) - t_1[(n_1 - n_0) + c(n_0 - n_2)] - c(t_0 - t_1)(n_1 - n_2)}{[(n_1 - n_2) - (n_1 - n_0) - c(n_0 - n_2)]} \tag{4-10}$$

As an example of the VSSI procedure, reconsider the filling operation of the previous sections. Recall that plastic bottles are filled with a liquid to a target value, $\mu_0 = 2000$ ml. The standard deviation of the process is stable and known to be $\sigma = 6$ ml. This process is susceptible to shifts in the process mean resulting in either over or under-filled bottles.

Previous monitoring with a standard Shewhart $\overline{X}$ chart required subgroups of 5 bottles taken once an hour. A VSSI adaptive scheme is desired, but the average inter-sample time is to remain at one hour and the average sample size should be 5 bottles. Since the VSSI scheme allows the sample size to vary, we will monitor the process using

a standardized chart as demonstrated in the previous section. The upper and lower chart limits will be ± 3.

Following the advice of Prabhu, Montgomery, and Runger we will fix the adaptive sample sizes and lower sampling interval. Due to space limitations and sample collection workload the maximum sample size is limited to 20 bottles. Management feels sample sizes should not drop below 2 bottles at any time. Therefore set $n_1 = 2$ and $n_2 = 20$. The threshold value is now determined using equation 4-9.

$$w = \Phi^{-1}\left[\frac{\Phi(3)(2)(5-20)+(2-5)}{2(2-20)}\right] = \Phi^{-1}(0.916) = 1.38$$

Constraints that affect the sampling interval center around the time required to pull a sample, measure each bottle, record the values and return them to the bottling line. The minimum practical interval between samples is estimated to be 15 minutes if the associated sample size is not overly large. Previous experience indicates the line should not operate for more than 2 hours between samples. Now set $t_1 = 0.25$ and determine t_2 using equation 4-10.

$$t_2 = \frac{t_0(n_1 - n_2) - t_1[(n_1 - n_0) + c(n_0 - n_2)] - c(t_0 - t_1)(n_1 - n_2)}{[(n_1 - n_2) - (n_1 - n_0) - c(n_0 - n_2)]}$$

where $c = \Phi(UCL)(2) = 1.9973$

$$t_2 = \frac{1(2-20) - 0.25[(2-5) + c(5-20)] - c(1-0.25)(2-20)}{[(2-20) - (2-5) - c(5-20)]} = 1.15 \text{ hours} = 1 \text{ hour, } 09 \text{ minutes}$$

Now we can establish upper and lower sampling thresholds of $\mu_0 + w = 1.38$, and -1.38 respectively. Figure 4-7 shows the standardized VSSI chart with threshold limits and how samples in each zone will affect the next sample taken.

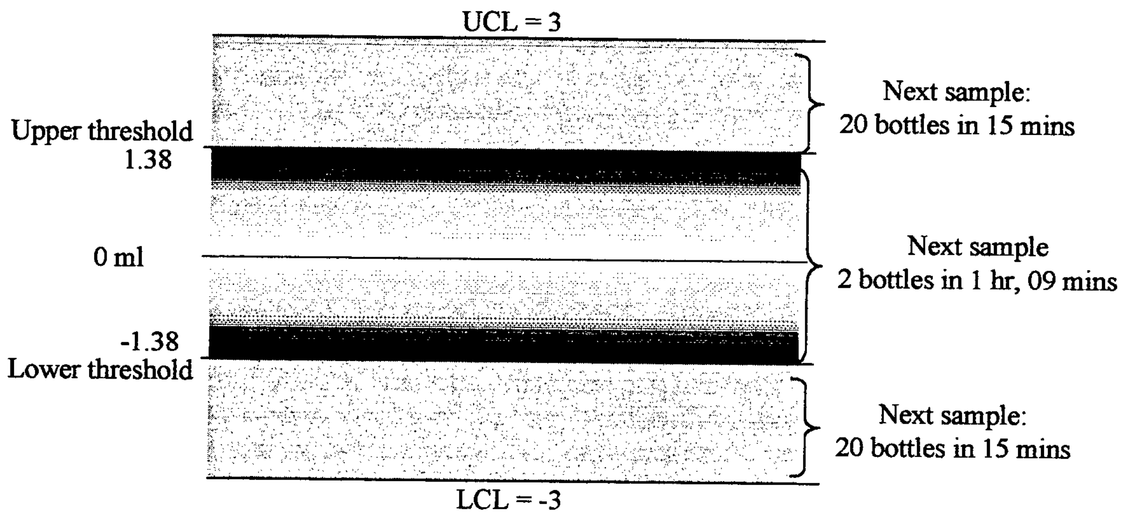


FIGURE 4-7. Standardized VSSI Chart Showing Threshold and Limit Values and Appropriate Actions

Assume the first sample from the bottling operation occurs at 8 o'clock a.m., $t(1) = 08{:}00$. Prabhu, Montgomery, and Runger suggest conducting the first sample as though the previous sample had been beyond the threshold limits. Initializing the chart procedure in this manner gives the chart the best opportunity to catch an off-target condition that might be present at start-up. If the first sample (size = 20) is within the chart limits, but outside the threshold limits, say $\overline{X}_1 = 1996.3$ (standardizing using equation 4-1 gives $Z_1 = -2.76$), the next sample should be taken at $t(2) = t(1) + t_1 = 08{:}15$. If the average of the next sample, $\overline{X}_2$ (that is the sample taken at 08:15), falls within the

threshold limits, say 2001.2, we would schedule the next sample at $t(3) = t(2) + 1{:}09 =$ 09:26.

This procedure continues until a sample mean exceeds the chart limits. The tables given by Prabhu, Montgomery, and Runger indicate that, if the process mean should shift by, say 1.5 σ to a new mean of 2000 + 2(9) = 2018, the chart will, on average, signal within 1.44 hours of the process shift. This is compared with 1.57 hours using a standard, fixed-interval Shewhart chart. This result may not be enough of an improvement to warrant the added complexity of the VSSI scheme. However, Table 4-2 shows that the VSSI approach significantly outperforms the standard Shewhart chart, the VSI chart, and the VSS chart for small shifts in the process mean. The differences for larger shift sizes can be minimized if we allow smaller values for both n_1 and n_2. The VSS data in Table 4-2 is from Prabhu, Runger, and Keats (1993), the VSI data from Prabhu, Montgomery, and Runger (1994). The VSSI data agrees with the numbers given by Prabhu, Montgomery, and Runger, but was obtained using the Markov chain ATS, and the modified probability of detection computer program found in the appendix to Chapter 3. This provided an opportunity to test the program before applying it to fractional sampling situations.

TABLE 4-2. Comparison of ARLs for Shewhart ($n_0 = 5$), VSS, VSI, and VSSI Schemes

	t_1	t_2	n_1	n_2	w	$\delta = 0$	$\delta = 0.5$	$\delta = 1.0$	$\delta = 2.0$
Shewhart chart						370.42	33.41	4.50	1.08
VSS chart									
			1	8	0.56	370.42	22.61	6.28	1.39
			1	12	0.91	370.42	15.34	2.57	1.62
			1	20	1.25	370.42	9.88	2.93	1.86
			2	8	0.67	370.42	23.06	2.90	1.29
			2	12	1.03	370.42	15.93	2.47	1.41
			2	20	1.38	370.42	10.29	2.59	1.51
			3	8	0.84	370.42	23.75	2.92	1.20
			3	12	1.22	370.42	17.01	2.44	1.25
			3	20	1.55	370.42	11.30	2.45	1.29
			4	8	1.15	370.42	25.14	2.98	1.12
			4	12	1.52	370.42	19.42	2.49	1.14
			4	20	1.85	370.42	13.97	2.46	1.15
VSI chart									
	0.25	2.00			0.56	370.42	23.34	2.23	1.02
	0.25	1.43			0.91	370.42	24.62	2.35	1.02
	0.25	1.20			1.25	370.42	26.30	2.56	1.02
	0.25	1.75			0.67	370.42	23.69	2.26	1.02
	0.25	1.32			1.03	370.42	25.21	2.42	1.02
	0.25	1.15			1.38	370.42	27.00	2.66	1.02
	0.25	1.50			0.84	370.42	24.33	2.32	1.02
	0.25	1.21			1.22	370.42	26.13	2.54	1.02
	0.25	1.10			1.55	370.42	27.98	2.82	1.02
	0.25	1.25			1.15	370.42	25.77	2.49	1.02
	0.25	1.11			1.52	370.42	27.82	2.79	1.02
	0.25	1.05			1.85	370.42	29.54	3.12	1.02
VSSI chart									
	0.25	2.00	1	8	0.56	370.42	15.28	1.82	1.15
	0.25	1.43	1	12	0.91	370.42	10.85	1.98	1.27
	0.25	1.20	1	20	1.25	370.42	7.74	2.54	1.43
	0.25	1.75	2	8	0.67	370.42	15.71	1.74	1.08
	0.25	1.32	2	12	1.03	370.42	11.39	1.78	1.13
	0.25	1.15	2	20	1.38	370.42	18.06	2.10	1.19
	0.25	1.50	3	8	0.84	370.42	16.57	1.72	1.05
	0.25	1.21	3	12	1.22	370.42	12.58	1.70	1.07
	0.25	1.10	3	20	1.55	370.42	19.07	1.93	1.09
	0.25	1.25	4	8	1.15	370.42	18.71	1.79	1.03
	0.25	1.11	4	12	1.52	370.42	15.50	1.76	1.04
	0.25	1.05	4	20	1.85	370.42	11.94	1.96	1.05

As discussed in the previous sections, we may be able to use the VSSI approach to simply improve the current monitoring situation. For example, set n_2 equal to the existing sample size and t_1 equal to the current inter-sample time, let n_1 approach a single unit and solve for t_2. We could then develop a new VSSI monitoring plan where the data is collected of the same size and at the same rate as the fixed rate plan when we exceed the threshold values, but at a more "leisurely" pace when we are within the threshold values. A note of caution here: In this case n_0 and t_0 will not equal the existing process values, but will instead be at least marginally larger than the small sample size and frequent sampling interval respectively.

Adaptive Multiple Stream Monitoring

Let's now consider how to apply the concepts of adaptive process monitoring to MSP situations. The idea of adapting the interval between samples is exactly the same as discussed in the previous sections. Two sampling intervals are chosen and a threshold value selected such that the average time between samples equals some desired fixed rate of sampling. The concept of adapting sample size can also be easily transferred to the MSP problem although how we define a sample will have an important impact on the results. Two sample size definitions will be presented. The first assumes all streams in the process are included in a sample, the second allows fractional sampling.

<u>Complete Stream Sampling.</u> Traditionally, multiple stream processes define a sample of size of one to be a sample of a single item from each stream. Using this definition, a sample size of 5 would involve 5 items from each stream. For example, in a

process with 10 streams, a sample of 5 would involve 50 production items. Adapting the sample size would therefore select between two sample sizes with an appropriate threshold value to result in an average sample size of a desired number of full collections from the MSP.

The combined adaptive approach of the VSSI technique applied to the MSP problem would simply involve a combination of the VSI and VSS approaches. Redefining the bottling process as a MSP will show how the adaptive techniques might work. The basics of the process remain the same. Plastic bottles are filled with a liquid to a target value ($\mu_0 = 2000$ ml), the standard deviation of each stream is stable and known to be equal to $\sigma = 6$ ml. Rather than being filled one at a time bottles are filled on a rotary-filling machine with 12 valves. In this case a sample size of 1 involves 12 bottles.

Past monitoring of this process has been accomplished with a fixed interval, fixed sample size Shewhart $\overline{X}$ chart using 3 samples from each valve (36 bottles) taken each hour. Monitoring each stream would require $p = 12$ charts with upper and lower chart limits of $UCL_{\overline{X}} = \mu_0 + 3\sigma/\sqrt{n} = 2000 + 3 \cdot 6/\sqrt{3} \cong 2010.39$ and $LCL_{\overline{X}} \cong 1989.61$. As we saw earlier, we can greatly simplify the monitoring situation by taking an average value across all the streams. Upper and lower limits for monitoring the average are $UCL_{\overline{\overline{X}}} = \mu_0 + 3\sigma_{\overline{X}}/\sqrt{p} = 2000 + 3 \cdot 3.464/\sqrt{12} = 2003$ and $LCL_{\overline{\overline{X}}} = 1997$. Given this background, a VSSI adaptive scheme is desired, with an average inter-sample time of one hour and an average sample size of 3.

Fixing the adaptive sample sizes and lower sampling interval will again define the threshold value and upper sampling interval. Due to space limitations and sample collection workload, the maximum sample size is limited to 60 bottles (5 turns of the filling machine). Management feels sample sizes should not drop below 1 bottle from each stream at any time. Therefore set $n_1 = 1$ and $n_2 = 5$. The threshold value is now determined using equation 4-9.

$$w = \Phi^{-1}\left[\frac{\Phi(3)(2)(3-5)+(1-3)}{2(1-5)}\right] = \Phi^{-1}(0.7493) = 0.6724$$

The sampling interval is constrained by the time required to pull a sample, measure each bottle, record the values and return them to the bottling line. The minimum practical interval between samples is estimated to be 15 minutes if the associated sample size is not overly large. Previous experience indicates the line should not operate for more than 2 hours between samples. Now set $t_1 = 0.25$ and determine t_2 using equation 4-10.

$$t_2 = \frac{1(1-5)-0.25[(1-3)+c(3-5)]-c(1-0.25)(1-5)}{[(1-5)-(1-3)-c(3-5)]} = 1.75 \text{ hours}$$

Now we can establish upper and lower sampling thresholds and chart limits. Table 4-3 shows the appropriate values for n_1 and n_2 in non-standardized units.

TABLE 4-3. Actual Limits for MSP VSSI Chart (n_1=1, n_2=5)

	LCL	*Lower Threshold*	*Upper Threshold*	*UCL*
$n_1 = 1$ (12 bottles)	1994.80	1998.84	2001.16	2005.20
$n_2 = 5$ (60 bottles)	1997.68	1999.48	2000.52	2002.32

To simplify things, we will use a standardized chart with threshold values of ±0.6724. Figure 4-8 shows the standardized VSSI chart with threshold limits and how samples in each zone will affect the next sample taken.

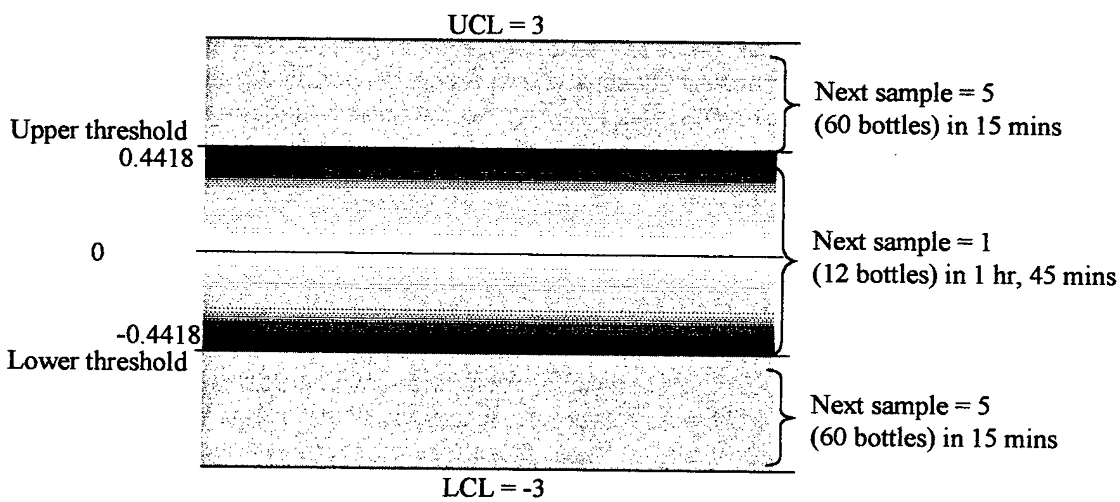


FIGURE 4-8. Standardized MSP VSSI Chart Showing Threshold and Limit Values and Appropriate Actions.

Now assume the first sample from the bottling operation occurs at 8 o'clock a.m., $t(1)$ = 08:00 with a sample size of 5 called for when using the FIR approach suggested by Prabhu, Montgomery, and Runger. If the first sample taken is within the chart limits, but outside the threshold limits, say $\overline{X}_1 = 1998.6$, the next sample should be taken at $t(2)$ =

$t(1) + t_1 = 08{:}15$ using a sample size of $n_2 = 5$. If the average of the next sample, $\bar{X}_2$ (that is the sample taken at 08:15), falls within the threshold limits, say 2000.2, we would schedule the next sample at $t(3) = t(2) + 1{:}45 = 10{:}00$ using a sample size of 1.

This procedure continues until a sample mean exceeds the chart limits. The tables given by Prabhu, Montgomery, and Runger indicate that, if the process mean shifts by, say 1.5 σ, to a new mean of 2000 + 1.5(6) = 2009 across all streams, the chart will signal within 1.26 hours on average as compared with 2.91 hours using a standard, fixed-interval Shewhart chart. Refer to Table 4-2 for performance comparisons of the VSSI approach compared to the standard Shewhart chart, the VSI chart, and the VSS chart.

Note that the preceding results are valid if we have p identically distributed, independent streams, all the streams shift together, and samples consist of full turns of the filling machine (all streams sampled). Recall at the beginning of this section we mentioned that the definition of a sample size would have a direct impact on how the VSSI approach was applied to a MSP situation. If, instead of taking samples consisting of data from every stream, we sample only a fraction of the streams at any given time, the definition of a sample size will be slightly different. This situation will be investigated next.

Fractional Stream Sampling. When faced with a fractional sampling situation, we will consider the number of streams sampled, s, to be the sample size. So if a process has 60 streams and 25% of the streams are sampled, the sample size is $s = 15$. Using a VSSI adaptive approach adapts the size of this fractional sample (s_1 and s_2) and the time

between fractional samples (t_1 and t_2). In the case where we were sampling from each stream, the $\overline{X}$ chart only gave an indication if an assignable cause had impacted all the streams and did a poor job of noting when individual streams had moved off-target. Earlier we saw how Mortell and Runger (1995) address this issue by also monitoring the range, R_t, of each sample. This measure performs well in detecting shifts affecting one or more streams. However, Mortell and Runger require the size of each sample taken to be equal, a problem if we intend to use a VSS or VSSI approach. To accommodate this requirement, only fractional samples where s_2 is an integer multiple of s_1 will be taken, and one or more range charts for each sample will be generated.

Since only a fraction of the total streams are sampled, a chart signal indicates a shift in the mean of that fraction, not necessarily all the streams in the process. The tables in Chapter 3 show that this signal is more likely the greater the number of streams in the sample that shift. For example, if we sample 20 of 60 streams at any given time, a signal indicates that a shift may be impacting the 20 streams sampled and a signal is more likely if 18 streams have shifted than if only 2 streams shift. So a signal indicates that either all streams in the process have shifted, or a significant number of the streams contained in the fractional sample. By adapting the sample fraction, we can quickly determine if the shift is affecting all streams or limited to a certain subset of the streams.

To clarify the fractional sampling VSSI approach, reconsider the MSP bottling process with a large number of streams. Once again we will assume plastic bottles are filled with a liquid to a target value ($\mu_0 = 2000$ ml), the standard deviation of each stream

is stable and known to be equal to $\sigma = 6$ ml. For this example, assume bottles are filled on a rotary-filling machine with 60 valves.

Past monitoring of this process has been accomplished with a fixed interval, fixed sample size Shewhart $\overline{X}$ chart using a sample from one third of the streams (20 bottles) every hour. By taking an average value across 20 streams, upper and lower limits have been established at $UCL_{\overline{X}} = \mu_0 + 3\sigma/\sqrt{s_0} = 2000 + 3 \cdot 6/\sqrt{20} = 2004.02$ and $LCL_{\overline{X}} = 1995.98$. A VSSI adaptive scheme is desired, but the average inter-sample time is to remain at one hour and the average sample size should be 20 bottles.

Again the adaptive sample sizes and lower sampling interval are fixed. In this case space limitations and sample collection workload limit the maximum sample size to 30 bottles. Since two fractional sample sizes where s_2 is an integer multiple of s_1 are desired, set $s_1 = 15$ and $s_2 = 30$. The threshold value can now be determined using equation 4-9.

$$w = \Phi^{-1}\left[\frac{\Phi(3)(2)(20-30)+(15-20)}{2(15-20)}\right] = \Phi^{-1}(0.8324) = 0.9638$$

Using a minimum interval between samples of 15 minutes ($t_1 = 0.25$) determine t_2 using equation 4-10.

$$t_2 = \frac{1(15-30)-0.25[(15-20)+c(20-30)]-c(1-0.25)(15-30)}{[(15-30)-(15-20)-c(20-30)]} = 1.375 \text{ hours}$$

Now upper and lower sampling thresholds and chart limits can be established. Table 4-4 shows the appropriate values for s_1 and s_2 in non-standardized units. A standardized chart with threshold values of ±0.9638 will be used to actually monitor the process for simplicity.

TABLE 4-4. Actual Limits for Fractional VSSI Chart (s_1=15, s_2=30)

	LCL	*Lower Threshold*	*Upper Threshold*	*UCL*
$s_1 = 15$	1991.95	1997.41	2002.59	2008.05
$s_2 = 30$	1996.71	1998.94	2001.06	2003.29

Now assume the first sample from the bottling operation occurs at 8 o'clock a.m., $t(1)$ = 08:00. The FIR approach requires the first sample to be s_2 = 30 bottles. If the first sample taken is within the $\overline{X}$ chart limits, but outside the threshold limits the next sample should be of size s_2 = 30 bottles taken at $t(2) = t(1) + t_1$ = 08:15. If the average of the next sample, $\overline{X}_2$ (that is the sample taken at 08:15), falls within the threshold limits we would schedule the next sample at $t(3) = t(2)$ + 1:22:30 = 09:37:30 using a sample size of s_1 = 15 bottles. This procedure continues until a sample mean exceeds the chart limits.

The tables given by Prabhu, Montgomery, and Runger do not help determine the chart's performance as we are sampling only a fraction of the total streams and allow a process shift to impact fewer than all the streams. Instead a Markov chain approach is

used to determine the performance of the adaptive, fractional $\overline{X}$ chart, in a fashion similar to that used by Prabhu, Montgomery, and Runger. The particulars of the Markov process are given in Appendix 4A.

The tables on the following pages show the ATS performance of various VSSI schemes for the 60 bottle process example using the Markov chain method of determining the ATS. Tables are given for average sample sizes of 12 and 24 bottles, and for situations with various numbers of off-target streams. Table 4-6, 4-7, 4-8 and 4-9 show results when 20%, 40%, 60%, and 80% of the streams are off-target respectively. Each table also shows the results of a fixed sample chart for the same situation. Note that the VSSI approach outperforms the fixed sample size approach in nearly all cases. Those instances where the fixed method holds an edge, it is seen to be only marginally better than the adaptive methods. Sets of tables for other MSP situations can be found at the end of the chapter.

TABLE 4-5. ATS Results for 12 Streams Off-target in a 60 Stream Process

s_0	s_1	s_2	t_1	t_2	w	*0*	*0.5*	*1*	*1.5*	*2*	*2.5*	*3*
12	fixed	–	–	–	–	370.38	199.24	64.54	23.03	10.34	5.76	3.80
12	6	18	0.25	1.75	0.67	370.38	187.64	50.12	14.29	5.59	3.07	2.16
12	6	24	0.25	1.38	0.96	370.38	186.83	47.50	12.72	4.90	2.79	2.06
12	6	30	0.25	1.25	1.15	370.38	185.98	45.21	11.48	4.43	2.64	2.03
12	6	36	0.25	1.19	1.28	370.38	185.10	43.14	10.48	4.11	2.57	2.05
12	6	42	0.25	1.15	1.38	370.38	184.20	41.24	9.67	3.90	2.56	2.08
12	6	48	0.25	1.13	1.46	370.38	183.28	39.49	9.01	3.77	2.58	2.14
12	6	54	0.25	1.11	1.52	370.38	182.37	37.87	8.46	3.70	2.62	2.19
12	6	60	0.25	1.09	1.58	370.38	181.46	36.36	8.02	3.67	2.67	2.25

s_0	s_1	s_2	t_1	t_2	w	*0*	*0.5*	*1*	*1.5*	*2*	*2.5*	*3*
24	fixed	–	–	–	–	370.38	145.88	36.98	12.03	5.29	3.02	2.08
24	6	30	0.25	3.25	0.32	370.38	130.42	24.34	6.02	2.54	1.67	1.38
24	6	36	0.25	2.13	0.52	370.38	128.21	22.18	5.25	2.33	1.64	1.41
24	6	42	0.25	1.75	0.67	370.38	126.24	20.39	4.70	2.22	1.65	1.45
24	6	48	0.25	1.56	0.79	370.38	124.40	18.85	4.29	2.16	1.68	1.50
24	6	54	0.25	1.45	0.88	370.38	122.65	17.52	3.98	2.14	1.72	1.55
24	6	60	0.25	1.38	0.96	370.38	120.99	16.36	3.76	2.15	1.77	1.61
24	12	30	0.25	2.50	0.43	370.38	131.03	24.65	6.09	2.55	1.66	1.35
24	12	36	0.25	1.75	0.67	370.38	129.56	22.84	5.39	2.33	1.61	1.36
24	12	42	0.25	1.50	0.84	370.38	128.24	21.33	4.86	2.21	1.59	1.37
24	12	48	0.25	1.38	0.96	370.38	127.00	20.00	4.46	2.13	1.60	1.40
24	12	54	0.25	1.30	1.06	370.38	125.80	18.83	4.16	2.09	1.61	1.43
24	12	60	0.25	1.25	1.15	370.38	124.63	17.78	3.92	2.07	1.64	1.46
24	18	30	0.25	1.75	0.67	370.38	132.46	25.50	6.34	2.61	1.67	1.35
24	18	36	0.25	1.38	0.96	370.38	132.26	24.47	5.84	2.44	1.62	1.34
24	18	42	0.25	1.25	1.15	370.38	131.94	23.52	5.42	2.32	1.60	1.35
24	18	48	0.25	1.19	1.28	370.38	131.52	22.63	5.07	2.24	1.59	1.36
24	18	54	0.25	1.15	1.38	370.38	131.04	21.78	4.78	2.19	1.60	1.38
24	18	60	0.25	1.13	1.46	370.38	130.52	20.98	4.55	2.16	1.61	1.40

TABLE 4-6. ATS Results for 24 Streams Off-target in a 60 Stream Process

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
12	fixed	–	–	–	–	370.38	81.27	14.27	4.52	2.32	1.61	1.32
12	6	18	0.25	1.75	0.67	370.38	62.57	6.54	2.08	1.43	1.24	1.16
12	6	24	0.25	1.38	0.96	370.38	58.38	5.42	1.97	1.46	1.29	1.20
12	6	30	0.25	1.25	1.15	370.38	54.79	4.76	1.96	1.51	1.34	1.24
12	6	36	0.25	1.19	1.28	370.38	51.62	4.37	2.00	1.57	1.38	1.27
12	6	42	0.25	1.15	1.38	370.38	48.78	4.15	2.07	1.62	1.42	1.30
12	6	48	0.25	1.13	1.46	370.38	46.22	4.03	2.14	1.67	1.45	1.32
12	6	54	0.25	1.11	1.52	370.38	43.92	4.00	2.22	1.72	1.48	1.34
12	6	60	0.25	1.09	1.58	370.38	41.83	4.01	2.29	1.77	1.50	1.36

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
24	fixed	–	–	–	–	370.38	42.25	6.04	2.09	1.30	1.09	1.03
24	6	30	0.25	3.25	0.32	370.38	27.18	2.57	1.33	1.14	1.09	1.06
24	6	36	0.25	2.13	0.52	370.38	24.35	2.33	1.37	1.20	1.14	1.10
24	6	42	0.25	1.75	0.67	370.38	22.06	2.22	1.42	1.25	1.18	1.13
24	6	48	0.25	1.56	0.79	370.38	20.16	2.18	1.47	1.30	1.21	1.16
24	6	54	0.25	1.45	0.88	370.38	18.56	2.18	1.53	1.34	1.24	1.18
24	6	60	0.25	1.38	0.96	370.38	17.20	2.20	1.58	1.38	1.27	1.20
24	12	30	0.25	2.50	0.43	370.38	27.59	2.57	1.30	1.11	1.05	1.03
24	12	36	0.25	1.75	0.67	370.38	25.18	2.32	1.30	1.13	1.07	1.04
24	12	42	0.25	1.50	0.84	370.38	23.20	2.19	1.32	1.15	1.09	1.05
24	12	48	0.25	1.38	0.96	370.38	21.51	2.12	1.35	1.18	1.10	1.06
24	12	54	0.25	1.30	1.06	370.38	20.05	2.08	1.37	1.20	1.11	1.07
24	12	60	0.25	1.25	1.15	370.38	18.78	2.08	1.40	1.21	1.12	1.07
24	18	30	0.25	1.75	0.67	370.38	28.66	2.65	1.29	1.09	1.03	1.01
24	18	36	0.25	1.38	0.96	370.38	27.20	2.43	1.28	1.10	1.04	1.02
24	18	42	0.25	1.25	1.15	370.38	25.87	2.30	1.29	1.11	1.05	1.02
24	18	48	0.25	1.19	1.28	370.38	24.66	2.22	1.30	1.12	1.05	1.02
24	18	54	0.25	1.15	1.38	370.38	23.53	2.18	1.32	1.13	1.06	1.03
24	18	60	0.25	1.13	1.46	370.38	22.50	2.16	1.34	1.14	1.06	1.03

TABLE 4-7. ATS Results for 36 Streams Off-target in a 60 Stream Process

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
12	fixed	–	–	–	–	370.38	35.99	5.01	1.85	1.24	1.08	1.03
12	6	18	0.25	1.75	0.67	370.38	20.17	2.05	1.26	1.11	1.06	1.04
12	6	24	0.25	1.38	0.96	370.38	16.91	1.93	1.31	1.15	1.09	1.05
12	6	30	0.25	1.25	1.15	370.38	14.60	1.93	1.36	1.19	1.11	1.06
12	6	36	0.25	1.19	1.28	370.38	12.91	1.99	1.41	1.21	1.12	1.07
12	6	42	0.25	1.15	1.38	370.38	11.66	2.06	1.45	1.23	1.13	1.08
12	6	48	0.25	1.13	1.46	370.38	10.72	2.15	1.49	1.25	1.14	1.08
12	6	54	0.25	1.11	1.52	370.38	10.03	2.24	1.53	1.27	1.15	1.09
12	6	60	0.25	1.09	1.58	370.38	9.51	2.32	1.56	1.28	1.15	1.09

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
24	fixed	–	–	–	–	370.38	15.06	2.09	1.12	1.01	1.00	1.00
24	6	30	0.25	3.25	0.32	370.38	6.72	1.31	1.09	1.05	1.03	1.02
24	6	36	0.25	2.13	0.52	370.38	5.65	1.35	1.14	1.08	1.05	1.03
24	6	42	0.25	1.75	0.67	370.38	4.96	1.40	1.18	1.10	1.06	1.04
24	6	48	0.25	1.56	0.79	370.38	4.49	1.46	1.22	1.12	1.07	1.04
24	6	54	0.25	1.45	0.88	370.38	4.17	1.51	1.25	1.14	1.08	1.05
24	6	60	0.25	1.38	0.96	370.38	3.96	1.57	1.28	1.15	1.09	1.05
24	12	30	0.25	2.50	0.43	370.38	6.80	1.28	1.06	1.02	1.01	1.00
24	12	36	0.25	1.75	0.67	370.38	5.81	1.28	1.08	1.03	1.01	1.00
24	12	42	0.25	1.50	0.84	370.38	5.13	1.30	1.09	1.03	1.01	1.00
24	12	48	0.25	1.38	0.96	370.38	4.65	1.32	1.11	1.04	1.01	1.01
24	12	54	0.25	1.30	1.06	370.38	4.31	1.35	1.12	1.04	1.01	1.01
24	12	60	0.25	1.25	1.15	370.38	4.07	1.38	1.13	1.05	1.02	1.01
24	18	30	0.25	1.75	0.67	370.38	7.15	1.27	1.04	1.01	1.00	1.00
24	18	36	0.25	1.38	0.96	370.38	6.39	1.26	1.05	1.01	1.00	1.00
24	18	42	0.25	1.25	1.15	370.38	5.82	1.27	1.06	1.01	1.00	1.00
24	18	48	0.25	1.19	1.28	370.38	5.37	1.28	1.06	1.01	1.00	1.00
24	18	54	0.25	1.15	1.38	370.38	5.03	1.30	1.07	1.01	1.00	1.00
24	18	60	0.25	1.13	1.46	370.38	4.77	1.32	1.07	1.01	1.00	1.00

TABLE 4-8. ATS Results for 48 Streams Off-target in a 60 Stream Process

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
12	fixed	–	–	–	–	370.38	17.85	2.41	1.18	1.02	1.00	1.00
12	6	18	0.25	1.75	0.67	370.38	7.24	1.34	1.09	1.03	1.01	1.00
12	6	24	0.25	1.38	0.96	370.38	5.76	1.38	1.13	1.05	1.02	1.01
12	6	30	0.25	1.25	1.15	370.38	4.97	1.44	1.15	1.06	1.02	1.01
12	6	36	0.25	1.19	1.28	370.38	4.54	1.50	1.17	1.06	1.02	1.01
12	6	42	0.25	1.15	1.38	370.38	4.32	1.56	1.19	1.07	1.02	1.01
12	6	48	0.25	1.13	1.46	370.38	4.24	1.61	1.21	1.07	1.03	1.01
12	6	54	0.25	1.11	1.52	370.38	4.23	1.66	1.22	1.08	1.03	1.01
12	6	60	0.25	1.09	1.58	370.38	4.29	1.71	1.23	1.08	1.03	1.01

s_0	s_1	s_2	t_1	t_2	w	0	0.5	1	1.5	2	2.5	3
24	fixed	–	–	–	–	370.38	6.58	1.23	1.00	1.00	1.00	1.00
24	6	30	0.25	3.25	0.32	370.38	2.59	1.12	1.04	1.02	1.01	1.00
24	6	36	0.25	2.13	0.52	370.38	2.34	1.17	1.07	1.03	1.01	1.00
24	6	42	0.25	1.75	0.67	370.38	2.22	1.21	1.09	1.03	1.01	1.00
24	6	48	0.25	1.56	0.79	370.38	2.19	1.25	1.10	1.04	1.01	1.01
24	6	54	0.25	1.45	0.88	370.38	2.20	1.29	1.11	1.04	1.02	1.01
24	6	60	0.25	1.38	0.96	370.38	2.24	1.32	1.13	1.05	1.02	1.01
24	12	30	0.25	2.50	0.43	370.38	2.58	1.08	1.01	1.00	1.00	1.00
24	12	36	0.25	1.75	0.67	370.38	2.31	1.10	1.02	1.00	1.00	1.00
24	12	42	0.25	1.50	0.84	370.38	2.17	1.12	1.02	1.00	1.00	1.00
24	12	48	0.25	1.38	0.96	370.38	2.10	1.14	1.03	1.00	1.00	1.00
24	12	54	0.25	1.30	1.06	370.38	2.08	1.15	1.03	1.00	1.00	1.00
24	12	60	0.25	1.25	1.15	370.38	2.08	1.17	1.03	1.00	1.00	1.00
24	18	30	0.25	1.75	0.67	370.38	2.66	1.07	1.00	1.00	1.00	1.00
24	18	36	0.25	1.38	0.96	370.38	2.43	1.07	1.01	1.00	1.00	1.00
24	18	42	0.25	1.25	1.15	370.38	2.29	1.08	1.01	1.00	1.00	1.00
24	18	48	0.25	1.19	1.28	370.38	2.21	1.09	1.01	1.00	1.00	1.00
24	18	54	0.25	1.15	1.38	370.38	2.17	1.10	1.01	1.00	1.00	1.00
24	18	60	0.25	1.13	1.46	370.38	2.16	1.10	1.01	1.00	1.00	1.00

In addition to comparisons with the fixed sample scheme, it is interesting to note performance differences among the VSSI schemes. Figure 4-9 shows how changes in s_2 affect the ATS performance for a process with 60 streams, 40 percent of which are off-target using an average sample size, $s_0 = 12$ and a minimum sample size, $s_1 = 6$. Note that while larger values of s_2 provide better results, they are not dramatically better than the smaller s_2 values.

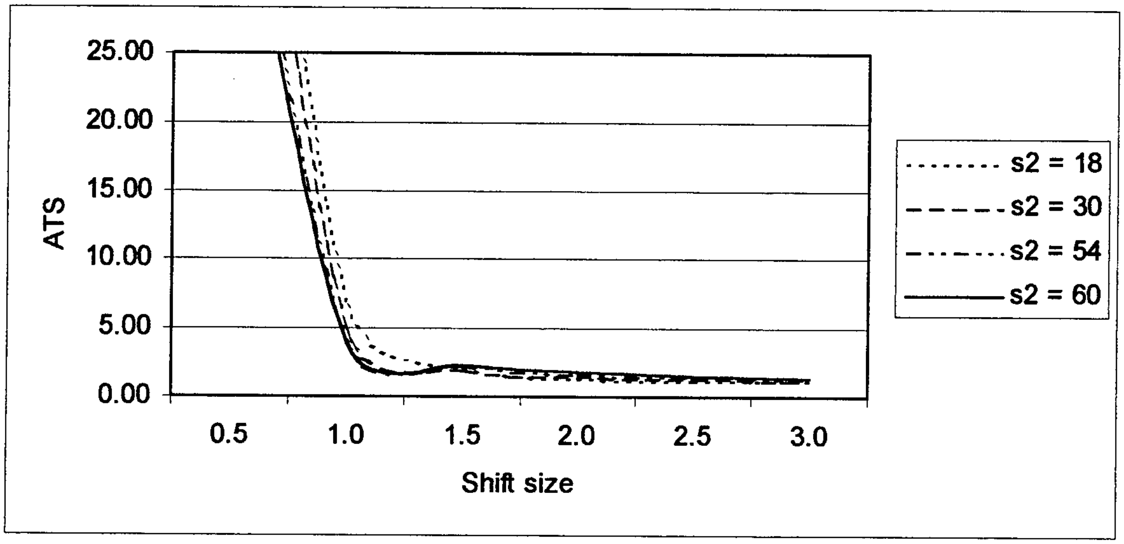


FIGURE 4-9. ATS Results for Various s_2 and t_2 Values in a 60 Stream Process with 40% Off-target and $s_0 = 12$, $s_1 = 6$, $t_0 = 1.0$, $t_1 = 0.25$

Figure 4-10 shows how increasing the average sample size, s_0, improves ATS performance for a 60 stream process with 40% off-target and a common threshold value, $w = 0.67$. In general, larger values of s_0 perform better, but smaller values of s_0 can be competitive performers.

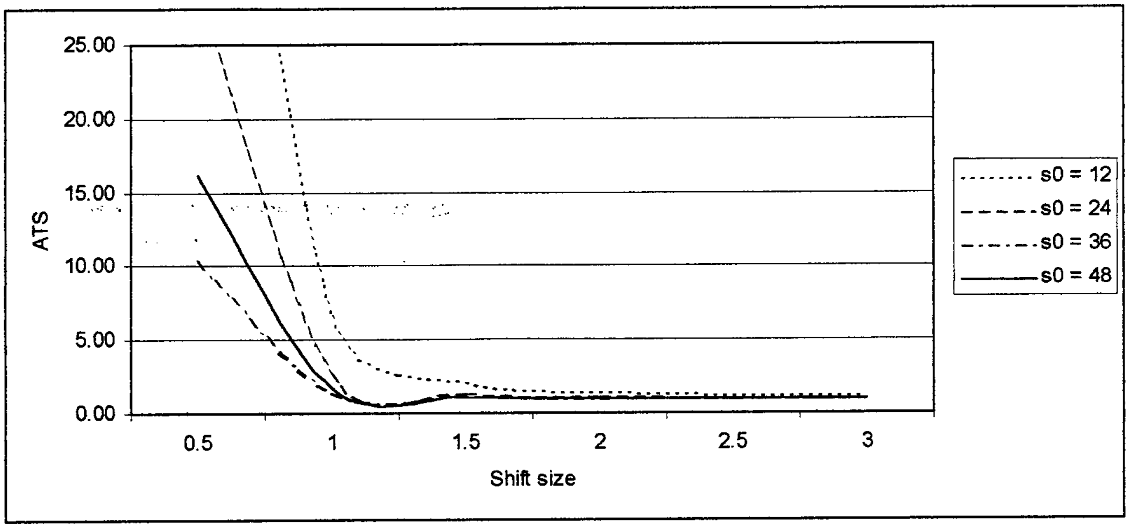


FIGURE 4-10. ATS Values for a 60 Stream Process with Various Average Sample Sizes, 40% Off-target, $w = 0.67$, $t_0 = 1.0$, $t_1 = 0.25$

Figure 4-11 shows how changes in the percentage of off-target (OT) streams impacts a VSSI scheme for 60 streams with an average sample size of $s_0 = 12$ and common threshold value of $w = 0.67$.

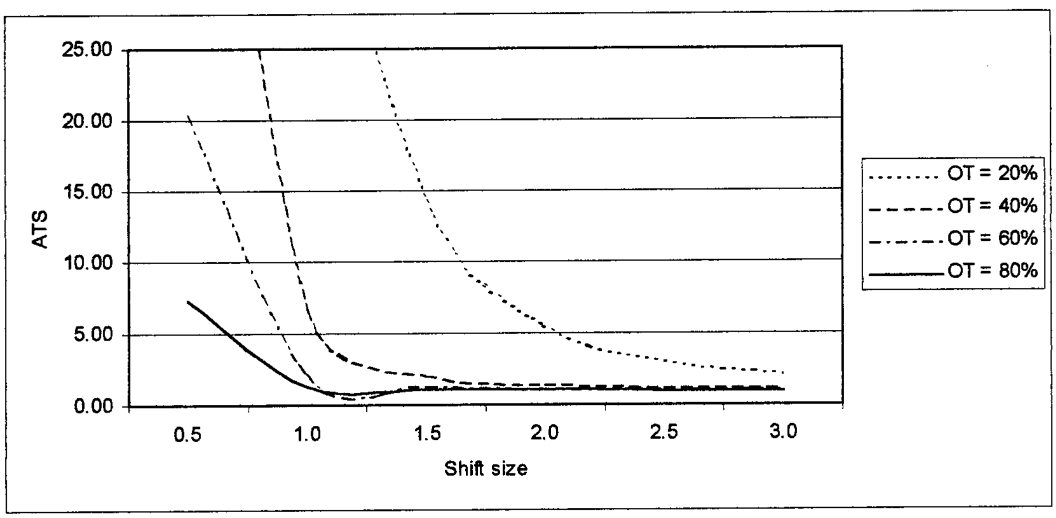


FIGURE 4-11. ATS Values for 60 Stream Process with Various Off-Target Percentages, $w = 0.67$, $s_0 = 12$, $s_1 = 6$, $s_2 = 18$, $t_0 = 1.0$, $t_1 = 0.25$, $t_2 = 1.75$

Selecting an adaptive scheme to use involves practical considerations beyond simply choosing the best ATS performance. In many cases the desired shift size to be detected affects the choice of the best VSSI scheme. Another important consideration centers on the threshold value, w. Adaptive schemes where w is less than 0.67 indicate more samples will fall in zone 2 than in zone 1 for an on-target process, thereby necessitating the taking of frequent large samples. As w is allowed to increase, the proportion of samples that fall in zone 1 will also increase (for an on-target process).

Range Monitoring

In addition to $\overline{X}$, the range for each fractional sample should also be monitored using the R_t chart. Since the range chart requires all samples to be of a fixed size, values for the variable sample size portion of an adaptive scheme should be chosen with care. By selecting the larger value, s_2, as an integer multiple of the smaller value, s_1, multiple ranges can be calculated when the larger sample size is taken. Each of these ranges should be plotted on the range chart in the order taken so that a signal will allow proper identification of the suspect sample fraction. In the previous bottling example, two ranges would be calculated when $s = s_2 = 30$: the first derived from the first 15 bottles, the second using bottles 16 – 30. This way all range chart values are based on a common sample size of 15 bottles. Furthermore, if the process has shifted, these range charts will help indicate which fraction contains the shift.

Appropriate range limits can be determined by estimating the expected fractional sample range when the process is on target. For a given sample size we can determine the probability of realizing a range value above a given limit by simulating the situation, or referring to table of Normal distribution range limits. Pearson (1932) provides a detailed table of sample range percentage limits for samples of size 2 to 100.

Note that the traditional purpose of the range chart is to track process variance, but here it is used to watch for a mean shift. Since the range chart is monitoring the maximum and minimum values within a sample, it is actually estimating the stream-to-stream variability. Any statistical chart that accomplishes the same task ought to serve as an adequate mechanism for monitoring shifts that impact a subset of the process streams.

Summary

The algorithm developed in Chapter 3 has allowed the performance of adaptive monitoring schemes for fractionally sampled multiple stream processes to be measured using a Markov chain approach. Specific performance results are dependent on the parameters of the process being monitored, but the results presented here show that the adaptive technique generally outperforms fixed sample size methods. Furthermore, in many cases the performance of an adaptively monitored process using fractional samples can compete directly with schemes where all streams are required to be included in a given sample.

The tables and graphs in Appendix 4B show a great deal of flexibility is available in determining an appropriate adaptive monitoring scheme. To illustrate the

implementation of an adaptive scheme for a process requiring fractional sampling, a case study will be presented in Chapter 5.

APPENDIX 4A

ATS DEVELOPMENT USING MARKOV CHAINS

The average time to signal (ATS) is used as a performance measure to indicate the expected value of the elapsed time between the occurrence of an assignable cause and its detection by a given process monitoring scheme. A large ATS is desired when the process is on-target, but a short ATS is needed when the process drifts off-target since we want to detect this situation as quickly as possible. We are interested in detecting situations where the process mean shifts from a target value, μ_0 to a new value, μ. The size of the shift is commonly expressed in terms of the process standard deviation, σ. Thus we desire to detect the situation where μ_0 shifts to $\mu = \mu_0 + \delta\sigma$.

A Markov chain technique can be used to determine the ATS. A Markov process is defined as a process independent of previous actions and dependent only on the current state of the process. That the adaptive process fits this description can be seen in that the probability of transitioning from one state to another is a function only of the present state and the applied sampling decision rule.

The development given in this appendix closely parallels the description given by Prabhu, Montgomery and Runger (1994) although the specifics given here will be unique to the fractionally sampled MSP. Other Markov approaches to determining average run lengths are given by Brook and Evans (1972), Prabhu, Runger, and Keats (1993), and Costa (1994).

A traditional Shewhart chart can be thought of as consisting of two zones, or states. The on-target state where plotted data points fall within the chart limits, and the off-target state, where a plotted point falls beyond the chart limits. By specifying the location of the chart limits, we define the probability of being in one state or the other for a given sample. In adaptive monitoring schemes, more than two states are defined for the system. For adaptive schemes using one set of threshold values and one set of chart limits, three possible states are defined.

State 1: within the threshold values = $[-w, w]$
State 2: outside the threshold values, but within the chart limits
$= [\text{LCL}, -w] \cup [w, \text{UCL}]$
State 3: outside the chart limits = $(-\infty, \text{LCL}] \cup [\text{UCL}, \infty)$

State 3 can be seen to be an *absorbing state*, that is no sampling action takes place if we enter state 3, rather we cease monitoring, stop the process, and search for an assignable cause which may have impacted the process.

The development of a Markov chain approach to determining the ATS begins with a transition probability matrix. For the adaptive models used in this chapter this matrix is given by

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

where p_{ij} represents the probability of moving from state i to state j for a given shift size in the mean, δ. For example, p_{11} represents the probability that being in state 1, we will remain in state 1 after taking a sample when the process has shifted by δ. Since we are currently in state 1 we will be using s_1 as the sample size and so $p_{11} = \Pr[-w < Z_i < w \mid s_1; \delta]$. Now solving using the cumulative standard normal distribution gives

$$p_{11} = \Phi\left(w - \delta\sqrt{s_1}\right) - \Phi\left(-w - \delta\sqrt{s_1}\right).$$

To determine the ARL for a VSS chart, Prabhu, Runger, and Keats (1993) and Costa (1994) use the following equation.

$$\text{ARL} = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} \tag{4A-1}$$

where $\mathbf{b}' = (b_1, b_2)$ is a vector relating to the probabilities that the process starts in state 1 and state 2 respectively. Since we do not allow the process to start in state 3, $b_1 + b_2 = 1$. $\mathbf{I}$ is the identity matrix of order 2, $\mathbf{Q}$ is the probability transition matrix where the elements associated with the absorbing state have been deleted, and $\mathbf{1}$ is a 2 x 1 column vector.

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Prabhu, Montgomery, and Runger point out that $\mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}$ provides the mean number of transitions in each state before the adaptive scheme signals. To account for the variable time between samples of the VSSI scheme, $\mathbf{1}$ is replaced by the vector of sampling intervals, $\mathbf{t}' = [t_2, t_1]$, in equation 4A -1 yielding

$$\text{ATS} = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{t} \tag{4A-2}$$

The choice of $\mathbf{b}'$ is critical to determining the ATS of the process. This vector represents the probability that the monitoring scheme will be in zone 1 or zone 2 (state1, or state 2) when the process shift occurs. Naturally if the adaptive scheme happens to be in zone 2 when the process shift occurs, there is a better chance of detecting the shift immediately due to the larger sample size, s_2, used in this zone. Prabhu, Montgomery, and Runger suggest reasonable choices for b_1 and b_2 as the proportion of time spent using in zones 1 and 2 respectively while the process is on target. Hence $b_1 = p_{11}/(p_{11}+p_{12})$ and $b_2 = p_{22}/(p_{21}+p_{22})$.

At this point the only information we need to compute the ATS are the transition probabilities. As we are taking fractional samples from a process where any number of the streams may be off-target, determining these probabilities can be quite involved. However, the program used in Chapter 3 to determine probabilities of detection is easily modified to obtain transition probabilities. This is the approach used to generate the results shown in Chapter 4 and the tables given in Appendix 4B.

Allowing the sample fraction to vary, as well as the sampling interval, results in an infinite number of possible combinations. Limiting the allowable values of t_1 to 0.25, 0.50, and 0.75 and incrementing s_1 and s_2 in steps of 10 percent of the total streams helps somewhat, but still results in more tables of results than can be shown in this document. In all cases, using values of $t_1 = 0.25$ provides the best ATS results. This is not surprising

as smaller values of t_1 result in larger values of t_2 and the spread between values is maximized resulting in better ATS performance as suggested by Reynolds (1989) and Reynolds and Arnold (1989).

Generally, smaller values of s_1 together with values of s_2 approaching 100 percent of the total streams provide better ATS values. Exceptions to this rule occur as the number of streams shifting off-target increases. For example, if 4 streams are off-target and $s_1 = 2$ streams, there is not as much opportunity to obtain off-target samples as if $s_1 =$ 4 or more streams, especially as the shift sizes increase. Table 4A-1 shows an example of this. Notice that while s_1 has increased for the larger shift sizes, s_2 has continued to prefer its maximum value, 100 percent of the total streams.

TABLE 4A-1. ATS Results for Fractional VSSI (s_0=16, t_1=0.25)

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	2	18	0.25	6.25	0.16	370.38	181.78	51.10	14.85	5.32	2.61	1.73
16	2	20	0.25	3.63	0.28	370.38	**181.20**	**49.86**	**13.99**	**4.88**	2.42	1.67
16	4	18	0.25	5.50	0.18	370.38	181.82	51.14	14.87	5.32	2.60	1.72
16	4	20	0.25	3.25	0.32	370.38	181.32	49.98	14.04	4.89	**2.41**	1.65
16	6	18	0.25	4.75	0.21	370.38	181.88	51.20	14.89	5.33	2.61	1.72
16	6	20	0.25	2.88	0.37	370.38	181.48	50.15	14.11	4.91	2.42	1.65
16	8	18	0.25	4.00	0.25	370.38	181.97	51.29	14.94	5.34	2.61	1.72
16	8	20	0.25	2.50	0.43	370.38	181.71	50.39	14.23	4.95	2.43	**1.64**
16	10	18	0.25	3.25	0.32	370.38	182.12	51.46	15.02	5.37	2.62	1.72
16	10	20	0.25	2.13	0.52	370.38	182.08	50.80	14.44	5.03	2.45	1.65
16	12	18	0.25	2.50	0.43	370.38	182.45	51.80	15.19	5.45	2.64	1.72
16	12	20	0.25	1.75	0.67	370.38	182.78	51.57	14.84	5.19	2.50	1.66
16	14	18	0.25	1.75	0.67	370.38	183.43	52.85	15.76	5.68	2.73	1.76
16	14	20	0.25	1.38	0.96	370.38	184.47	53.48	15.89	5.63	2.66	1.71

Table 4A-2 shows the average time to signal for the adaptive approach with an average sample size of $s_0 = 20$ bottles, compared with a fixed sample approach using $n =$ 20 bottles. The ATS values for the adaptive method were obtained using $s_1 = 15$ and $s_2 =$

30, $t_1 = 0.25$ and $t_2 = 1.38$, and $w = 0.964$ and are shown in bold on the left with the corresponding fixed sample results on the right. Results are shown for various numbers of streams shifting by sizes ranging from 0.5σ to 3σ for each method.

TABLE 4A-2. ATS Results for Fractional VSSI ($s_1 = 15$, $s_2 = 30$, $s_0 = 20$) vs. Fixed Sample ($n = 20$) Schemes

	Shift Size											
# Off-target	***0.5***		***1.0***		***1.5***		***2.0***		***2.5***		***3.0***	
12	**148.0**	160.5	**30.8**	43.5	**7.7**	14.5	**3.1**	6.4	**1.9**	3.6	**1.5**	2.5
24	**35.4**	51.1	**3.1**	7.7	**1.4**	2.5	**1.2**	1.5	**1.1**	1.2	**1.0**	1.1
36	**9.0**	19.2	**1.4**	2.6	**1.1**	1.2	**1.0**	1.0	**1.0**	1.0	**1.0**	1.0
48	**3.2**	8.6	**1.1**	1.4	**1.0**	1.0	**1.0**	1.0	**1.0**	1.0	**1.0**	1.0
60	**1.8**	4.5	**1.0**	1.1	**1.0**	1.0	**1.0**	1.0	**1.0**	1.0	**1.0**	1.0

APPENDIX 4B

ATS TABLES FOR 40 & 80 STREAM PROCESSES

The tables shown in this appendix were derived using the Markov chain approach described in Appendix 4A. Tables are given here for processes with 40 streams and 80 streams.

40 Streams; 8 Streams Off-target; s_0 = 8 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
8	4	12	0.25	1.75	0.67	370.38	218.86	71.97	23.17	9.24	4.80	3.12
8	4	16	0.25	1.38	0.96	370.38	218.80	70.22	21.65	8.36	4.33	2.87
8	4	20	0.25	1.25	1.15	370.38	218.64	68.63	20.34	7.65	3.99	2.72
8	4	24	0.25	1.19	1.28	370.38	218.39	67.12	19.18	7.07	3.75	2.64
8	4	28	0.25	1.15	1.38	370.38	218.10	65.68	18.13	6.59	3.58	2.60
8	4	32	0.25	1.13	1.46	370.38	217.78	64.30	17.19	6.20	3.46	2.59
8	4	36	0.25	1.11	1.52	370.38	217.43	62.97	16.32	5.88	3.39	2.61
8	4	40	0.25	1.09	1.58	370.38	217.08	61.70	15.54	5.62	3.36	2.64

40 Streams; 16 Streams Off-target; s_0 = 8 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
8	4	12	0.25	1.75	0.67	370.38	93.66	12.56	3.32	1.84	1.44	1.29
8	4	16	0.25	1.38	0.96	370.38	90.33	10.71	2.93	1.79	1.47	1.33
8	4	20	0.25	1.25	1.15	370.38	87.32	9.40	2.74	1.80	1.52	1.38
8	4	24	0.25	1.19	1.28	370.38	84.53	8.43	2.66	1.85	1.57	1.43
8	4	28	0.25	1.15	1.38	370.38	81.91	7.72	2.66	1.91	1.62	1.47
8	4	32	0.25	1.13	1.46	370.38	79.44	7.19	2.69	1.97	1.67	1.50
8	4	36	0.25	1.11	1.52	370.38	77.10	6.80	2.75	2.03	1.72	1.53
8	4	40	0.25	1.09	1.58	370.38	74.89	6.52	2.82	2.09	1.76	1.56

40 Streams; 24 Streams Off-target; s_0 = 8 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
8	4	12	0.25	1.75	0.67	370.38	37.58	3.42	1.52	1.22	1.13	1.08
8	4	16	0.25	1.38	0.96	370.38	33.41	2.94	1.53	1.27	1.17	1.12
8	4	20	0.25	1.25	1.15	370.38	30.09	2.74	1.58	1.32	1.20	1.14
8	4	24	0.25	1.19	1.28	370.38	27.38	2.69	1.64	1.36	1.23	1.16
8	4	28	0.25	1.15	1.38	370.38	25.12	2.70	1.70	1.40	1.25	1.17
8	4	32	0.25	1.13	1.46	370.38	23.24	2.76	1.76	1.43	1.27	1.18
8	4	36	0.25	1.11	1.52	370.38	21.65	2.84	1.82	1.46	1.28	1.19
8	4	40	0.25	1.09	1.58	370.38	20.32	2.94	1.88	1.49	1.30	1.20

40 Streams; 36 Streams Off-target; s_0 = 8 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
8	4	12	0.25	1.75	0.67	370.38	15.39	1.74	1.18	1.08	1.04	1.02
8	4	16	0.25	1.38	0.96	370.38	12.55	1.69	1.23	1.11	1.05	1.03
8	4	20	0.25	1.25	1.15	370.38	10.67	1.73	1.27	1.13	1.06	1.03
8	4	24	0.25	1.19	1.28	370.38	9.40	1.80	1.31	1.15	1.07	1.04
8	4	28	0.25	1.15	1.38	370.38	8.52	1.88	1.35	1.16	1.08	1.04
8	4	32	0.25	1.13	1.46	370.38	7.91	1.96	1.38	1.17	1.08	1.04
8	4	36	0.25	1.11	1.52	370.38	7.50	2.04	1.41	1.18	1.09	1.04
8	4	40	0.25	1.09	1.58	370.38	7.24	2.12	1.44	1.19	1.09	1.05

40 Streams; 8 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	4	20	0.25	3.25	0.32	370.38	168.81	41.37	11.31	4.34	2.41	1.75
16	4	24	0.25	2.13	0.52	370.38	167.55	39.19	10.16	3.87	2.24	1.70
16	4	28	0.25	1.75	0.67	370.38	166.45	37.28	9.22	3.52	2.14	1.69
16	4	32	0.25	1.56	0.79	370.38	165.42	35.58	8.44	3.27	2.08	1.71
16	4	36	0.25	1.45	0.88	370.38	164.44	34.02	7.78	3.09	2.07	1.74
16	4	40	0.25	1.38	0.96	370.38	163.50	32.59	7.22	2.96	2.07	1.78
16	8	20	0.25	2.50	0.43	370.38	169.28	41.76	11.45	4.38	2.42	1.74
16	8	24	0.25	1.75	0.67	370.38	168.59	40.05	10.45	3.94	2.24	1.68
16	8	28	0.25	1.50	0.84	370.38	167.98	38.57	9.63	3.62	2.13	1.65
16	8	32	0.25	1.38	0.96	370.38	167.40	37.22	8.93	3.37	2.06	1.65
16	8	36	0.25	1.30	1.06	370.38	166.82	35.97	8.32	3.18	2.03	1.66
16	8	40	0.25	1.25	1.15	370.38	166.24	34.80	7.80	3.03	2.01	1.68
16	12	20	0.25	1.75	0.67	370.38	170.45	42.79	11.87	4.53	2.47	1.76
16	12	24	0.25	1.38	0.96	370.38	170.75	42.05	11.26	4.22	2.33	1.70
16	12	28	0.25	1.25	1.15	370.38	170.87	41.32	10.70	3.95	2.23	1.67
16	12	32	0.25	1.19	1.28	370.38	170.86	40.58	10.18	3.74	2.16	1.66
16	12	36	0.25	1.15	1.38	370.38	170.77	39.84	9.71	3.55	2.11	1.66
16	12	40	0.25	1.13	1.46	370.38	170.62	39.11	9.26	3.40	2.08	1.67

40 Streams; 16 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	4	20	0.25	3.25	0.32	370.38	47.82	4.73	1.69	1.27	1.15	1.10
16	4	24	0.25	2.13	0.52	370.38	44.58	4.08	1.66	1.31	1.21	1.16
16	4	28	0.25	1.75	0.67	370.38	41.81	3.66	1.67	1.37	1.26	1.20
16	4	32	0.25	1.56	0.79	370.38	39.37	3.38	1.70	1.42	1.31	1.24
16	4	36	0.25	1.45	0.88	370.38	37.21	3.20	1.75	1.47	1.35	1.27
16	4	40	0.25	1.38	0.96	370.38	35.26	3.09	1.80	1.52	1.39	1.30
16	8	20	0.25	2.50	0.43	370.38	48.39	4.77	1.67	1.23	1.11	1.07
16	8	24	0.25	1.75	0.67	370.38	45.78	4.16	1.61	1.25	1.14	1.09
16	8	28	0.25	1.50	0.84	370.38	43.53	3.75	1.60	1.27	1.17	1.11
16	8	32	0.25	1.38	0.96	370.38	41.52	3.46	1.60	1.30	1.19	1.13
16	8	36	0.25	1.30	1.06	370.38	39.70	3.26	1.62	1.33	1.21	1.14
16	8	40	0.25	1.25	1.15	370.38	38.02	3.12	1.65	1.35	1.23	1.15
16	12	20	0.25	1.75	0.67	370.38	49.75	4.98	1.69	1.22	1.10	1.05
16	12	24	0.25	1.38	0.96	370.38	48.42	4.51	1.63	1.22	1.11	1.06
16	12	28	0.25	1.25	1.15	370.38	47.15	4.16	1.60	1.24	1.12	1.07
16	12	32	0.25	1.19	1.28	370.38	45.91	3.89	1.60	1.25	1.13	1.08
16	12	36	0.25	1.15	1.38	370.38	44.71	3.68	1.60	1.27	1.14	1.08
16	12	40	0.25	1.13	1.46	370.38	43.55	3.53	1.62	1.29	1.15	1.09

40 Streams; 24 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	4	20	0.25	3.25	0.32	370.38	13.97	1.67	1.17	1.08	1.05	1.04
16	4	24	0.25	2.13	0.52	370.38	12.00	1.64	1.22	1.13	1.09	1.06
16	4	28	0.25	1.75	0.67	370.38	10.54	1.66	1.28	1.17	1.11	1.08
16	4	32	0.25	1.56	0.79	370.38	9.42	1.70	1.32	1.20	1.13	1.09
16	4	36	0.25	1.45	0.88	370.38	8.55	1.75	1.37	1.23	1.15	1.11
16	4	40	0.25	1.38	0.96	370.38	7.88	1.81	1.41	1.25	1.17	1.12
16	8	20	0.25	2.50	0.43	370.38	14.19	1.65	1.13	1.05	1.02	1.01
16	8	24	0.25	1.75	0.67	370.38	12.43	1.59	1.15	1.07	1.03	1.02
16	8	28	0.25	1.50	0.84	370.38	11.09	1.57	1.18	1.08	1.04	1.02
16	8	32	0.25	1.38	0.96	370.38	10.03	1.58	1.20	1.09	1.05	1.02
16	8	36	0.25	1.30	1.06	370.38	9.19	1.60	1.22	1.10	1.05	1.03
16	8	40	0.25	1.25	1.15	370.38	8.50	1.63	1.24	1.11	1.05	1.03
16	12	20	0.25	1.75	0.67	370.38	14.88	1.66	1.12	1.03	1.01	1.00
16	12	24	0.25	1.38	0.96	370.38	13.66	1.60	1.12	1.04	1.01	1.00
16	12	28	0.25	1.25	1.15	370.38	12.64	1.57	1.14	1.04	1.01	1.00
16	12	32	0.25	1.19	1.28	370.38	11.76	1.57	1.15	1.05	1.02	1.01
16	12	36	0.25	1.15	1.38	370.38	11.01	1.58	1.16	1.05	1.02	1.01
16	12	40	0.25	1.13	1.46	370.38	10.37	1.60	1.17	1.06	1.02	1.01

40 Streams; 32 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	4	20	0.25	3.25	0.32	370.38	4.97	1.23	1.07	1.03	1.02	1.01
16	4	24	0.25	2.13	0.52	370.38	4.21	1.28	1.11	1.06	1.03	1.02
16	4	28	0.25	1.75	0.67	370.38	3.74	1.33	1.14	1.07	1.04	1.02
16	4	32	0.25	1.56	0.79	370.38	3.45	1.38	1.17	1.09	1.04	1.02
16	4	36	0.25	1.45	0.88	370.38	3.28	1.43	1.20	1.10	1.05	1.03
16	4	40	0.25	1.38	0.96	370.38	3.19	1.48	1.22	1.11	1.05	1.03
16	8	20	0.25	2.50	0.43	370.38	5.02	1.19	1.04	1.01	1.00	1.00
16	8	24	0.25	1.75	0.67	370.38	4.29	1.20	1.05	1.01	1.00	1.00
16	8	28	0.25	1.50	0.84	370.38	3.82	1.23	1.07	1.02	1.00	1.00
16	8	32	0.25	1.38	0.96	370.38	3.51	1.25	1.07	1.02	1.00	1.00
16	8	36	0.25	1.30	1.06	370.38	3.31	1.27	1.08	1.02	1.01	1.00
16	8	40	0.25	1.25	1.15	370.38	3.18	1.30	1.09	1.02	1.01	1.00
16	12	20	0.25	1.75	0.67	370.38	5.27	1.18	1.02	1.00	1.00	1.00
16	12	24	0.25	1.38	0.96	370.38	4.68	1.18	1.03	1.00	1.00	1.00
16	12	28	0.25	1.25	1.15	370.38	4.27	1.19	1.03	1.00	1.00	1.00
16	12	32	0.25	1.19	1.28	370.38	3.97	1.20	1.03	1.00	1.00	1.00
16	12	36	0.25	1.15	1.38	370.38	3.75	1.22	1.04	1.00	1.00	1.00
16	12	40	0.25	1.13	1.46	370.38	3.60	1.23	1.04	1.00	1.00	1.00

40 Streams; 8 Streams Off-target; s_0 = 24 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
24	4	28	0.25	4.75	0.21	370.38	135.52	27.01	6.70	2.67	1.67	1.35
24	4	32	0.25	2.88	0.37	370.38	133.97	25.31	6.01	2.46	1.63	1.37
24	4	36	0.25	2.25	0.49	370.38	132.56	23.81	5.45	2.31	1.62	1.40
24	4	40	0.25	1.94	0.59	370.38	131.25	22.47	5.00	2.22	1.63	1.44
24	8	28	0.25	4.00	0.25	370.38	135.70	27.10	6.72	2.67	1.66	1.33
24	8	32	0.25	2.50	0.43	370.38	134.38	25.52	6.05	2.45	1.61	1.34
24	8	36	0.25	2.00	0.56	370.38	133.21	24.15	5.51	2.30	1.58	1.35
24	8	40	0.25	1.75	0.67	370.38	132.14	22.92	5.07	2.20	1.58	1.38
24	12	28	0.25	3.25	0.32	370.38	135.95	27.25	6.75	2.67	1.65	1.32
24	12	32	0.25	2.13	0.52	370.38	134.97	25.87	6.14	2.46	1.60	1.32
24	12	36	0.25	1.75	0.67	370.38	134.13	24.68	5.64	2.31	1.56	1.32
24	12	40	0.25	1.56	0.79	370.38	133.36	23.62	5.22	2.20	1.55	1.33
24	16	28	0.25	2.50	0.43	370.38	136.40	27.54	6.84	2.69	1.66	1.32
24	16	32	0.25	1.75	0.67	370.38	135.97	26.52	6.33	2.50	1.60	1.31
24	16	36	0.25	1.50	0.84	370.38	135.60	25.64	5.90	2.36	1.56	1.30
24	16	40	0.25	1.38	0.96	370.38	135.24	24.83	5.54	2.26	1.54	1.31
24	20	28	0.25	1.75	0.67	370.38	137.63	28.39	7.14	2.78	1.68	1.32
24	20	32	0.25	1.38	0.96	370.38	138.20	28.13	6.87	2.66	1.64	1.31
24	20	36	0.25	1.25	1.15	370.38	138.53	27.80	6.61	2.55	1.61	1.31
24	20	40	0.25	1.19	1.28	370.38	138.69	27.43	6.35	2.45	1.58	1.31

40 Streams; 16 Streams Off-target; s_0 = 24 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
24	4	28	0.25	4.75	0.21	370.38	29.06	2.71	1.32	1.13	1.08	1.06
24	4	32	0.25	2.88	0.37	370.38	26.83	2.48	1.35	1.18	1.13	1.10
24	4	36	0.25	2.25	0.49	370.38	24.93	2.34	1.39	1.23	1.18	1.14
24	4	40	0.25	1.94	0.59	370.38	23.27	2.26	1.44	1.28	1.22	1.17
24	8	28	0.25	4.00	0.25	370.38	29.18	2.70	1.30	1.10	1.05	1.03
24	8	32	0.25	2.50	0.43	370.38	27.11	2.47	1.31	1.13	1.08	1.06
24	8	36	0.25	2.00	0.56	370.38	25.33	2.31	1.33	1.16	1.11	1.07
24	8	40	0.25	1.75	0.67	370.38	23.78	2.21	1.35	1.19	1.13	1.09
24	12	28	0.25	3.25	0.32	370.38	29.37	2.71	1.29	1.09	1.04	1.02
24	12	32	0.25	2.13	0.52	370.38	27.53	2.47	1.28	1.11	1.05	1.03
24	12	36	0.25	1.75	0.67	370.38	25.96	2.31	1.29	1.12	1.07	1.04
24	12	40	0.25	1.56	0.79	370.38	24.58	2.21	1.30	1.14	1.08	1.05
24	16	28	0.25	2.50	0.43	370.38	29.73	2.73	1.28	1.08	1.03	1.01
24	16	32	0.25	1.75	0.67	370.38	28.31	2.51	1.27	1.09	1.04	1.02
24	16	36	0.25	1.50	0.84	370.38	27.09	2.36	1.27	1.10	1.04	1.02
24	16	40	0.25	1.38	0.96	370.38	25.99	2.26	1.27	1.11	1.05	1.02
24	20	28	0.25	1.75	0.67	370.38	30.72	2.83	1.28	1.07	1.02	1.01
24	20	32	0.25	1.38	0.96	370.38	30.19	2.68	1.27	1.08	1.03	1.01
24	20	36	0.25	1.25	1.15	370.38	29.61	2.56	1.27	1.08	1.03	1.01
24	20	40	0.25	1.19	1.28	370.38	29.00	2.47	1.28	1.09	1.03	1.01

40 Streams; 24 Streams Off-target; s_0 = 24 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
24	4	28	0.25	4.75	0.21	370.38	7.27	1.32	1.09	1.05	1.03	1.02
24	4	32	0.25	2.88	0.37	370.38	6.36	1.35	1.14	1.09	1.06	1.04
24	4	36	0.25	2.25	0.49	370.38	5.68	1.39	1.18	1.12	1.08	1.06
24	4	40	0.25	1.94	0.59	370.38	5.18	1.44	1.23	1.15	1.10	1.07
24	8	28	0.25	4.00	0.25	370.38	7.28	1.29	1.06	1.02	1.01	1.01
24	8	32	0.25	2.50	0.43	370.38	6.39	1.30	1.09	1.04	1.02	1.01
24	8	36	0.25	2.00	0.56	370.38	5.73	1.32	1.11	1.05	1.03	1.01
24	8	40	0.25	1.75	0.67	370.38	5.22	1.34	1.13	1.06	1.03	1.02
24	12	28	0.25	3.25	0.32	370.38	7.33	1.28	1.05	1.01	1.00	1.00
24	12	32	0.25	2.13	0.52	370.38	6.49	1.27	1.06	1.02	1.01	1.00
24	12	36	0.25	1.75	0.67	370.38	5.86	1.28	1.07	1.03	1.01	1.00
24	12	40	0.25	1.56	0.79	370.38	5.37	1.29	1.09	1.03	1.01	1.00
24	16	28	0.25	2.50	0.43	370.38	7.44	1.27	1.04	1.01	1.00	1.00
24	16	32	0.25	1.75	0.67	370.38	6.72	1.26	1.05	1.01	1.00	1.00
24	16	36	0.25	1.50	0.84	370.38	6.16	1.26	1.05	1.01	1.00	1.00
24	16	40	0.25	1.38	0.96	370.38	5.71	1.26	1.06	1.01	1.00	1.00
24	20	28	0.25	1.75	0.67	370.38	7.82	1.27	1.03	1.00	1.00	1.00
24	20	32	0.25	1.38	0.96	370.38	7.38	1.26	1.04	1.00	1.00	1.00
24	20	36	0.25	1.25	1.15	370.38	6.99	1.26	1.04	1.01	1.00	1.00
24	20	40	0.25	1.19	1.28	370.38	6.64	1.26	1.04	1.01	1.00	1.00

40 Streams; 32 Streams Off-target; s_0 = 24 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
24	4	28	0.25	4.75	0.21	370.38	2.74	1.11	1.04	1.02	1.01	1.01
24	4	32	0.25	2.88	0.37	370.38	2.50	1.16	1.08	1.04	1.02	1.01
24	4	36	0.25	2.25	0.49	370.38	2.36	1.21	1.10	1.05	1.03	1.01
24	4	40	0.25	1.94	0.59	370.38	2.29	1.26	1.13	1.06	1.03	1.02
24	8	28	0.25	4.00	0.25	370.38	2.73	1.09	1.02	1.01	1.00	1.00
24	8	32	0.25	2.50	0.43	370.38	2.47	1.11	1.03	1.01	1.00	1.00
24	8	36	0.25	2.00	0.56	370.38	2.32	1.13	1.04	1.01	1.00	1.00
24	8	40	0.25	1.75	0.67	370.38	2.22	1.16	1.05	1.01	1.00	1.00
24	12	28	0.25	3.25	0.32	370.38	2.73	1.07	1.01	1.00	1.00	1.00
24	12	32	0.25	2.13	0.52	370.38	2.47	1.09	1.02	1.00	1.00	1.00
24	12	36	0.25	1.75	0.67	370.38	2.31	1.10	1.02	1.00	1.00	1.00
24	12	40	0.25	1.56	0.79	370.38	2.21	1.11	1.02	1.00	1.00	1.00
24	16	28	0.25	2.50	0.43	370.38	2.75	1.06	1.00	1.00	1.00	1.00
24	16	32	0.25	1.75	0.67	370.38	2.52	1.07	1.01	1.00	1.00	1.00
24	16	36	0.25	1.50	0.84	370.38	2.36	1.08	1.01	1.00	1.00	1.00
24	16	40	0.25	1.38	0.96	370.38	2.26	1.09	1.01	1.00	1.00	1.00
24	20	28	0.25	1.75	0.67	370.38	2.86	1.06	1.00	1.00	1.00	1.00
24	20	32	0.25	1.38	0.96	370.38	2.69	1.06	1.00	1.00	1.00	1.00
24	20	36	0.25	1.25	1.15	370.38	2.56	1.07	1.00	1.00	1.00	1.00
24	20	40	0.25	1.19	1.28	370.38	2.47	1.07	1.00	1.00	1.00	1.00

40 Streams; 8 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	4	36	0.25	6.25	0.16	370.38	111.74	18.89	4.44	1.94	1.37	1.19
32	4	40	0.25	3.63	0.28	370.38	110.16	17.62	4.03	1.85	1.38	1.22
32	8	36	0.25	5.50	0.18	370.38	111.83	18.92	4.44	1.93	1.36	1.17
32	8	40	0.25	3.25	0.32	370.38	110.38	17.69	4.03	1.84	1.36	1.20
32	12	36	0.25	4.75	0.21	370.38	111.94	18.96	4.44	1.93	1.35	1.17
32	12	40	0.25	2.88	0.37	370.38	110.64	17.79	4.04	1.83	1.34	1.18
32	16	36	0.25	4.00	0.25	370.38	112.09	19.02	4.45	1.93	1.34	1.16
32	16	40	0.25	2.50	0.43	370.38	111.01	17.95	4.07	1.83	1.33	1.17
32	20	36	0.25	3.25	0.32	370.38	112.32	19.13	4.48	1.93	1.34	1.15
32	20	40	0.25	2.13	0.52	370.38	111.56	18.22	4.13	1.83	1.32	1.15
32	24	36	0.25	2.50	0.43	370.38	112.76	19.37	4.54	1.94	1.34	1.15
32	24	40	0.25	1.75	0.67	370.38	112.54	18.74	4.26	1.85	1.32	1.15
32	28	36	0.25	1.75	0.67	370.38	114.02	20.09	4.75	1.99	1.35	1.15
32	28	40	0.25	1.38	0.96	370.38	114.79	20.07	4.63	1.93	1.33	1.15

40 Streams; 16 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	4	36	0.25	6.25	0.16	370.38	19.35	1.94	1.18	1.08	1.05	1.04
32	4	40	0.25	3.63	0.28	370.38	17.84	1.86	1.22	1.13	1.10	1.08
32	8	36	0.25	5.50	0.18	370.38	19.39	1.93	1.16	1.06	1.03	1.02
32	8	40	0.25	3.25	0.32	370.38	17.93	1.84	1.19	1.09	1.06	1.04
32	12	36	0.25	4.75	0.21	370.38	19.44	1.93	1.15	1.04	1.02	1.01
32	12	40	0.25	2.88	0.37	370.38	18.04	1.83	1.17	1.06	1.04	1.02
32	16	36	0.25	4.00	0.25	370.38	19.51	1.92	1.14	1.04	1.01	1.01
32	16	40	0.25	2.50	0.43	370.38	18.22	1.82	1.15	1.05	1.02	1.01
32	20	36	0.25	3.25	0.32	370.38	19.64	1.92	1.14	1.03	1.01	1.00
32	20	40	0.25	2.13	0.52	370.38	18.51	1.83	1.14	1.04	1.01	1.01
32	24	36	0.25	2.50	0.43	370.38	19.90	1.93	1.14	1.03	1.01	1.00
32	24	40	0.25	1.75	0.67	370.38	19.07	1.85	1.13	1.03	1.01	1.00
32	28	36	0.25	1.75	0.67	370.38	20.67	1.98	1.14	1.02	1.00	1.00
32	28	40	0.25	1.38	0.96	370.38	20.50	1.93	1.13	1.03	1.01	1.00

40 Streams; 24 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	4	36	0.25	6.25	0.16	370.38	4.53	1.18	1.06	1.04	1.03	1.02
32	4	40	0.25	3.63	0.28	370.38	4.08	1.22	1.10	1.07	1.05	1.03
32	8	36	0.25	5.50	0.18	370.38	4.53	1.16	1.04	1.02	1.01	1.00
32	8	40	0.25	3.25	0.32	370.38	4.07	1.18	1.06	1.03	1.02	1.01
32	12	36	0.25	4.75	0.21	370.38	4.53	1.15	1.02	1.01	1.00	1.00
32	12	40	0.25	2.88	0.37	370.38	4.08	1.16	1.04	1.01	1.00	1.00
32	16	36	0.25	4.00	0.25	370.38	4.54	1.14	1.02	1.00	1.00	1.00
32	16	40	0.25	2.50	0.43	370.38	4.10	1.15	1.03	1.01	1.00	1.00
32	20	36	0.25	3.25	0.32	370.38	4.56	1.14	1.01	1.00	1.00	1.00
32	20	40	0.25	2.13	0.52	370.38	4.16	1.14	1.02	1.00	1.00	1.00
32	24	36	0.25	2.50	0.43	370.38	4.63	1.13	1.01	1.00	1.00	1.00
32	24	40	0.25	1.75	0.67	370.38	4.29	1.13	1.01	1.00	1.00	1.00
32	28	36	0.25	1.75	0.67	370.38	4.85	1.13	1.01	1.00	1.00	1.00
32	28	40	0.25	1.38	0.96	370.38	4.69	1.13	1.01	1.00	1.00	1.00

40 Streams; 32 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	4	36	0.25	6.25	0.16	370.38	1.95	1.07	1.03	1.02	1.01	1.00
32	4	40	0.25	3.63	0.28	370.38	1.87	1.11	1.06	1.03	1.02	1.01
32	8	36	0.25	5.50	0.18	370.38	1.93	1.05	1.01	1.00	1.00	1.00
32	8	40	0.25	3.25	0.32	370.38	1.85	1.07	1.03	1.01	1.00	1.00
32	12	36	0.25	4.75	0.21	370.38	1.93	1.04	1.01	1.00	1.00	1.00
32	12	40	0.25	2.88	0.37	370.38	1.83	1.05	1.01	1.00	1.00	1.00
32	16	36	0.25	4.00	0.25	370.38	1.92	1.03	1.00	1.00	1.00	1.00
32	16	40	0.25	2.50	0.43	370.38	1.82	1.04	1.00	1.00	1.00	1.00
32	20	36	0.25	3.25	0.32	370.38	1.92	1.03	1.00	1.00	1.00	1.00
32	20	40	0.25	2.13	0.52	370.38	1.82	1.03	1.00	1.00	1.00	1.00
32	24	36	0.25	2.50	0.43	370.38	1.93	1.02	1.00	1.00	1.00	1.00
32	24	40	0.25	1.75	0.67	370.38	1.84	1.03	1.00	1.00	1.00	1.00
32	28	36	0.25	1.75	0.67	370.38	1.98	1.02	1.00	1.00	1.00	1.00
32	28	40	0.25	1.38	0.96	370.38	1.93	1.02	1.00	1.00	1.00	1.00

80 Streams; 16 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	8	24	0.25	1.75	0.67	370.38	162.56	36.47	9.58	3.84	2.29	1.74
16	8	32	0.25	1.38	0.96	370.38	160.98	33.55	8.24	3.37	2.15	1.72
16	8	40	0.25	1.25	1.15	370.38	159.44	31.11	7.29	3.11	2.11	1.74
16	8	48	0.25	1.19	1.28	370.38	157.92	28.99	6.61	2.98	2.12	1.79
16	8	56	0.25	1.15	1.38	370.38	156.41	27.13	6.10	2.93	2.16	1.85
16	8	64	0.25	1.13	1.46	370.38	154.93	25.50	5.74	2.92	2.22	1.90
16	8	72	0.25	1.11	1.52	370.38	153.46	24.05	5.49	2.96	2.28	1.96
16	8	80	0.25	1.09	1.58	370.38	152.02	22.76	5.31	3.01	2.35	2.02

80 Streams; 32 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	8	24	0.25	1.75	0.67	370.38	44.09	4.10	1.64	1.27	1.15	1.10
16	8	32	0.25	1.38	0.96	370.38	39.82	3.46	1.63	1.32	1.20	1.14
16	8	40	0.25	1.25	1.15	370.38	36.33	3.16	1.67	1.37	1.24	1.16
16	8	48	0.25	1.19	1.28	370.38	33.40	3.03	1.73	1.42	1.27	1.18
16	8	56	0.25	1.15	1.38	370.38	30.90	3.01	1.80	1.46	1.29	1.20
16	8	64	0.25	1.13	1.46	370.38	28.76	3.04	1.87	1.50	1.32	1.21
16	8	72	0.25	1.11	1.52	370.38	26.92	3.10	1.93	1.53	1.34	1.23
16	8	80	0.25	1.09	1.58	370.38	25.33	3.19	2.00	1.57	1.35	1.24

80 Streams; 48 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	8	24	0.25	1.75	0.67	370.38	12.20	1.60	1.16	1.07	1.03	1.02
16	8	32	0.25	1.38	0.96	370.38	9.86	1.59	1.21	1.10	1.05	1.03
16	8	40	0.25	1.25	1.15	370.38	8.39	1.64	1.25	1.12	1.06	1.03
16	8	48	0.25	1.19	1.28	370.38	7.43	1.71	1.28	1.13	1.06	1.03
16	8	56	0.25	1.15	1.38	370.38	6.80	1.78	1.31	1.14	1.07	1.04
16	8	64	0.25	1.13	1.46	370.38	6.40	1.85	1.34	1.15	1.08	1.04
16	8	72	0.25	1.11	1.52	370.38	6.15	1.92	1.37	1.16	1.08	1.04
16	8	80	0.25	1.09	1.58	370.38	6.01	1.99	1.39	1.17	1.08	1.04

80 Streams; 64 Streams Off-target; s_0 = 16 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
16	8	24	0.25	1.75	0.67	370.38	4.27	1.21	1.05	1.02	1.00	1.00
16	8	32	0.25	1.38	0.96	370.38	3.51	1.25	1.08	1.02	1.01	1.00
16	8	40	0.25	1.25	1.15	370.38	3.19	1.30	1.09	1.03	1.01	1.00
16	8	48	0.25	1.19	1.28	370.38	3.07	1.35	1.10	1.03	1.01	1.00
16	8	56	0.25	1.15	1.38	370.38	3.07	1.39	1.11	1.03	1.01	1.00
16	8	64	0.25	1.13	1.46	370.38	3.13	1.43	1.12	1.03	1.01	1.00
16	8	72	0.25	1.11	1.52	370.38	3.22	1.46	1.13	1.03	1.01	1.00
16	8	80	0.25	1.09	1.58	370.38	3.33	1.49	1.13	1.03	1.01	1.00

80 Streams; 16 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	8	40	0.25	3.25	0.32	370.38	103.63	15.72	3.84	1.88	1.40	1.24
32	8	48	0.25	2.13	0.52	370.38	100.77	13.89	3.37	1.80	1.43	1.29
32	8	56	0.25	1.75	0.67	370.38	98.21	12.45	3.07	1.79	1.47	1.35
32	8	64	0.25	1.56	0.79	370.38	95.85	11.28	2.88	1.80	1.52	1.40
32	8	72	0.25	1.45	0.88	370.38	93.63	10.32	2.77	1.84	1.58	1.45
32	8	80	0.25	1.38	0.96	370.38	91.54	9.53	2.71	1.88	1.63	1.50
32	16	40	0.25	2.50	0.43	370.38	104.32	15.94	3.87	1.87	1.38	1.21
32	16	48	0.25	1.75	0.67	370.38	102.26	14.35	3.42	1.77	1.37	1.23
32	16	56	0.25	1.50	0.84	370.38	100.42	13.07	3.12	1.73	1.39	1.25
32	16	64	0.25	1.38	0.96	370.38	98.71	12.01	2.92	1.72	1.41	1.28
32	16	72	0.25	1.30	1.06	370.38	97.07	11.12	2.78	1.73	1.44	1.31
32	16	80	0.25	1.25	1.15	370.38	95.50	10.35	2.70	1.75	1.47	1.34
32	24	40	0.25	1.75	0.67	370.38	105.87	16.60	4.02	1.89	1.37	1.19
32	24	48	0.25	1.38	0.96	370.38	105.24	15.58	3.67	1.80	1.36	1.20
32	24	56	0.25	1.25	1.15	370.38	104.54	14.68	3.41	1.76	1.36	1.21
32	24	64	0.25	1.19	1.28	370.38	103.78	13.87	3.21	1.74	1.38	1.23
32	24	72	0.25	1.15	1.38	370.38	102.97	13.14	3.07	1.74	1.39	1.25
32	24	80	0.25	1.13	1.46	370.38	102.15	12.48	2.97	1.74	1.41	1.26

80 Streams; 32 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	8	40	0.25	3.25	0.32	370.38	17.11	1.86	1.20	1.10	1.06	1.04
32	8	48	0.25	2.13	0.52	370.38	14.87	1.78	1.26	1.15	1.10	1.07
32	8	56	0.25	1.75	0.67	370.38	13.15	1.77	1.31	1.19	1.13	1.09
32	8	64	0.25	1.56	0.79	370.38	11.81	1.80	1.36	1.23	1.15	1.11
32	8	72	0.25	1.45	0.88	370.38	10.74	1.85	1.41	1.26	1.17	1.12
32	8	80	0.25	1.38	0.96	370.38	9.88	1.90	1.46	1.29	1.19	1.13
32	16	40	0.25	2.50	0.43	370.38	17.39	1.84	1.17	1.06	1.03	1.01
32	16	48	0.25	1.75	0.67	370.38	15.41	1.74	1.19	1.08	1.04	1.02
32	16	56	0.25	1.50	0.84	370.38	13.86	1.70	1.21	1.10	1.05	1.03
32	16	64	0.25	1.38	0.96	370.38	12.61	1.69	1.23	1.11	1.06	1.03
32	16	72	0.25	1.30	1.06	370.38	11.58	1.71	1.26	1.13	1.06	1.03
32	16	80	0.25	1.25	1.15	370.38	10.73	1.73	1.28	1.14	1.07	1.04
32	24	40	0.25	1.75	0.67	370.38	18.19	1.86	1.15	1.04	1.01	1.00
32	24	48	0.25	1.38	0.96	370.38	16.86	1.77	1.16	1.05	1.02	1.01
32	24	56	0.25	1.25	1.15	370.38	15.72	1.72	1.17	1.06	1.02	1.01
32	24	64	0.25	1.19	1.28	370.38	14.73	1.71	1.18	1.06	1.02	1.01
32	24	72	0.25	1.15	1.38	370.38	13.85	1.71	1.20	1.07	1.02	1.01
32	24	80	0.25	1.13	1.46	370.38	13.08	1.72	1.21	1.07	1.03	1.01

80 Streams; 48 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	8	40	0.25	3.25	0.32	370.38	4.03	1.19	1.06	1.03	1.02	1.01
32	8	48	0.25	2.13	0.52	370.38	3.46	1.24	1.10	1.05	1.03	1.01
32	8	56	0.25	1.75	0.67	370.38	3.13	1.29	1.13	1.07	1.03	1.02
32	8	64	0.25	1.56	0.79	370.38	2.94	1.34	1.16	1.08	1.04	1.02
32	8	72	0.25	1.45	0.88	370.38	2.85	1.39	1.18	1.09	1.04	1.02
32	8	80	0.25	1.38	0.96	370.38	2.81	1.44	1.20	1.10	1.05	1.03
32	16	40	0.25	2.50	0.43	370.38	4.06	1.15	1.03	1.01	1.00	1.00
32	16	48	0.25	1.75	0.67	370.38	3.50	1.17	1.04	1.01	1.00	1.00
32	16	56	0.25	1.50	0.84	370.38	3.16	1.19	1.05	1.01	1.00	1.00
32	16	64	0.25	1.38	0.96	370.38	2.95	1.22	1.06	1.02	1.00	1.00
32	16	72	0.25	1.30	1.06	370.38	2.82	1.24	1.07	1.02	1.00	1.00
32	16	80	0.25	1.25	1.15	370.38	2.74	1.26	1.07	1.02	1.00	1.00
32	24	40	0.25	1.75	0.67	370.38	4.25	1.14	1.02	1.00	1.00	1.00
32	24	48	0.25	1.38	0.96	370.38	3.79	1.14	1.02	1.00	1.00	1.00
32	24	56	0.25	1.25	1.15	370.38	3.48	1.15	1.02	1.00	1.00	1.00
32	24	64	0.25	1.19	1.28	370.38	3.26	1.17	1.03	1.00	1.00	1.00
32	24	72	0.25	1.15	1.38	370.38	3.12	1.18	1.03	1.00	1.00	1.00
32	24	80	0.25	1.13	1.46	370.38	3.02	1.19	1.03	1.00	1.00	1.00

80 Streams; 64 Streams Off-target; s_0 = 32 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
32	8	40	0.25	3.25	0.32	370.38	1.85	1.07	1.03	1.01	1.00	1.00
32	8	48	0.25	2.13	0.52	370.38	1.77	1.12	1.04	1.01	1.00	1.00
32	8	56	0.25	1.75	0.67	370.38	1.77	1.15	1.05	1.02	1.00	1.00
32	8	64	0.25	1.56	0.79	370.38	1.80	1.18	1.06	1.02	1.00	1.00
32	8	72	0.25	1.45	0.88	370.38	1.85	1.21	1.07	1.02	1.01	1.00
32	8	80	0.25	1.38	0.96	370.38	1.91	1.24	1.08	1.02	1.01	1.00
32	16	40	0.25	2.50	0.43	370.38	1.82	1.04	1.00	1.00	1.00	1.00
32	16	48	0.25	1.75	0.67	370.38	1.72	1.06	1.01	1.00	1.00	1.00
32	16	56	0.25	1.50	0.84	370.38	1.68	1.07	1.01	1.00	1.00	1.00
32	16	64	0.25	1.38	0.96	370.38	1.67	1.09	1.01	1.00	1.00	1.00
32	16	72	0.25	1.30	1.06	370.38	1.69	1.09	1.01	1.00	1.00	1.00
32	16	80	0.25	1.25	1.15	370.38	1.72	1.10	1.01	1.00	1.00	1.00
32	24	40	0.25	1.75	0.67	370.38	1.85	1.03	1.00	1.00	1.00	1.00
32	24	48	0.25	1.38	0.96	370.38	1.75	1.03	1.00	1.00	1.00	1.00
32	24	56	0.25	1.25	1.15	370.38	1.70	1.04	1.00	1.00	1.00	1.00
32	24	64	0.25	1.19	1.28	370.38	1.69	1.04	1.00	1.00	1.00	1.00
32	24	72	0.25	1.15	1.38	370.38	1.69	1.05	1.00	1.00	1.00	1.00
32	24	80	0.25	1.13	1.46	370.38	1.71	1.05	1.00	1.00	1.00	1.00

80 Streams; 16 Streams Off-target; s_0 = 48 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
48	8	56	0.25	4.75	0.21	370.38	72.63	8.71	2.31	1.41	1.19	1.12
48	8	64	0.25	2.88	0.37	370.38	70.04	7.73	2.16	1.42	1.24	1.17
48	8	72	0.25	2.25	0.49	370.38	67.70	6.95	2.07	1.45	1.29	1.23
48	8	80	0.25	1.94	0.59	370.38	65.55	6.33	2.02	1.49	1.34	1.27
48	16	56	0.25	4.00	0.25	370.38	72.85	8.74	2.31	1.39	1.17	1.09
48	16	64	0.25	2.50	0.43	370.38	70.53	7.78	2.14	1.38	1.20	1.12
48	16	72	0.25	2.00	0.56	370.38	68.46	7.03	2.04	1.39	1.22	1.15
48	16	80	0.25	1.75	0.67	370.38	66.55	6.43	1.97	1.41	1.25	1.18
48	24	56	0.25	3.25	0.32	370.38	73.15	8.79	2.30	1.38	1.15	1.08
48	24	64	0.25	2.13	0.52	370.38	71.22	7.91	2.14	1.36	1.17	1.09
48	24	72	0.25	1.75	0.67	370.38	69.52	7.21	2.03	1.36	1.18	1.11
48	24	80	0.25	1.56	0.79	370.38	67.96	6.63	1.96	1.37	1.20	1.13
48	32	56	0.25	2.50	0.43	370.38	73.67	8.92	2.32	1.37	1.14	1.07
48	32	64	0.25	1.75	0.67	370.38	72.38	8.18	2.16	1.35	1.15	1.08
48	32	72	0.25	1.50	0.84	370.38	71.24	7.58	2.05	1.34	1.16	1.09
48	32	80	0.25	1.38	0.96	370.38	70.16	7.07	1.98	1.34	1.17	1.10
48	40	56	0.25	1.75	0.67	370.38	75.03	9.34	2.39	1.38	1.14	1.06
48	40	64	0.25	1.38	0.96	370.38	74.94	8.93	2.28	1.36	1.14	1.07
48	40	72	0.25	1.25	1.15	370.38	74.69	8.55	2.19	1.35	1.15	1.07
48	40	80	0.25	1.19	1.28	370.38	74.34	8.18	2.13	1.35	1.16	1.08

80 Streams; 32 Streams Off-target; s_0 = 48 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
48	8	56	0.25	4.75	0.21	370.38	9.02	1.40	1.10	1.06	1.04	1.03
48	8	64	0.25	2.88	0.37	370.38	7.91	1.42	1.16	1.10	1.07	1.05
48	8	72	0.25	2.25	0.49	370.38	7.07	1.45	1.20	1.13	1.09	1.07
48	8	80	0.25	1.94	0.59	370.38	6.42	1.50	1.25	1.16	1.11	1.08
48	16	56	0.25	4.00	0.25	370.38	9.04	1.37	1.07	1.03	1.02	1.01
48	16	64	0.25	2.50	0.43	370.38	7.97	1.37	1.10	1.05	1.03	1.01
48	16	72	0.25	2.00	0.56	370.38	7.15	1.38	1.13	1.06	1.03	1.02
48	16	80	0.25	1.75	0.67	370.38	6.51	1.40	1.15	1.08	1.04	1.02
48	24	56	0.25	3.25	0.32	370.38	9.10	1.36	1.06	1.02	1.01	1.00
48	24	64	0.25	2.13	0.52	370.38	8.10	1.34	1.08	1.03	1.01	1.00
48	24	72	0.25	1.75	0.67	370.38	7.33	1.34	1.09	1.03	1.01	1.00
48	24	80	0.25	1.56	0.79	370.38	6.71	1.35	1.10	1.04	1.01	1.01
48	32	56	0.25	2.50	0.43	370.38	9.24	1.35	1.05	1.01	1.00	1.00
48	32	64	0.25	1.75	0.67	370.38	8.39	1.33	1.06	1.02	1.00	1.00
48	32	72	0.25	1.50	0.84	370.38	7.72	1.32	1.07	1.02	1.00	1.00
48	32	80	0.25	1.38	0.96	370.38	7.17	1.32	1.08	1.02	1.00	1.00
48	40	56	0.25	1.75	0.67	370.38	9.70	1.36	1.05	1.01	1.00	1.00
48	40	64	0.25	1.38	0.96	370.38	9.21	1.34	1.05	1.01	1.00	1.00
48	40	72	0.25	1.25	1.15	370.38	8.76	1.33	1.05	1.01	1.00	1.00
48	40	80	0.25	1.19	1.28	370.38	8.35	1.33	1.06	1.01	1.00	1.00

80 Streams; 48 Streams Off-target; s_0 = 48 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
48	8	56	0.25	4.75	0.21	370.38	2.33	1.10	1.04	1.02	1.01	1.01
48	8	64	0.25	2.88	0.37	370.38	2.17	1.15	1.07	1.04	1.02	1.01
48	8	72	0.25	2.25	0.49	370.38	2.09	1.19	1.09	1.05	1.02	1.01
48	8	80	0.25	1.94	0.59	370.38	2.05	1.24	1.11	1.06	1.03	1.02
48	16	56	0.25	4.00	0.25	370.38	2.31	1.07	1.02	1.00	1.00	1.00
48	16	64	0.25	2.50	0.43	370.38	2.14	1.09	1.03	1.01	1.00	1.00
48	16	72	0.25	2.00	0.56	370.38	2.03	1.12	1.04	1.01	1.00	1.00
48	16	80	0.25	1.75	0.67	370.38	1.98	1.14	1.04	1.01	1.00	1.00
48	24	56	0.25	3.25	0.32	370.38	2.31	1.06	1.01	1.00	1.00	1.00
48	24	64	0.25	2.13	0.52	370.38	2.13	1.07	1.01	1.00	1.00	1.00
48	24	72	0.25	1.75	0.67	370.38	2.02	1.08	1.02	1.00	1.00	1.00
48	24	80	0.25	1.56	0.79	370.38	1.95	1.10	1.02	1.00	1.00	1.00
48	32	56	0.25	2.50	0.43	370.38	2.32	1.05	1.00	1.00	1.00	1.00
48	32	64	0.25	1.75	0.67	370.38	2.15	1.06	1.00	1.00	1.00	1.00
48	32	72	0.25	1.50	0.84	370.38	2.04	1.06	1.01	1.00	1.00	1.00
48	32	80	0.25	1.38	0.96	370.38	1.97	1.07	1.01	1.00	1.00	1.00
48	40	56	0.25	1.75	0.67	370.38	2.40	1.04	1.00	1.00	1.00	1.00
48	40	64	0.25	1.38	0.96	370.38	2.28	1.05	1.00	1.00	1.00	1.00
48	40	72	0.25	1.25	1.15	370.38	2.19	1.05	1.00	1.00	1.00	1.00
48	40	80	0.25	1.19	1.28	370.38	2.12	1.05	1.00	1.00	1.00	1.00

80 Streams; 64 Streams Off-target; s_0 = 48 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
48	8	56	0.25	4.75	0.21	370.38	1.39	1.05	1.02	1.01	1.00	1.00
48	8	64	0.25	2.88	0.37	370.38	1.41	1.08	1.03	1.01	1.00	1.00
48	8	72	0.25	2.25	0.49	370.38	1.45	1.11	1.04	1.01	1.00	1.00
48	8	80	0.25	1.94	0.59	370.38	1.50	1.13	1.05	1.01	1.00	1.00
48	16	56	0.25	4.00	0.25	370.38	1.36	1.02	1.00	1.00	1.00	1.00
48	16	64	0.25	2.50	0.43	370.38	1.36	1.04	1.00	1.00	1.00	1.00
48	16	72	0.25	2.00	0.56	370.38	1.37	1.05	1.01	1.00	1.00	1.00
48	16	80	0.25	1.75	0.67	370.38	1.39	1.06	1.01	1.00	1.00	1.00
48	24	56	0.25	3.25	0.32	370.38	1.35	1.01	1.00	1.00	1.00	1.00
48	24	64	0.25	2.13	0.52	370.38	1.33	1.02	1.00	1.00	1.00	1.00
48	24	72	0.25	1.75	0.67	370.38	1.33	1.02	1.00	1.00	1.00	1.00
48	24	80	0.25	1.56	0.79	370.38	1.33	1.03	1.00	1.00	1.00	1.00
48	32	56	0.25	2.50	0.43	370.38	1.35	1.01	1.00	1.00	1.00	1.00
48	32	64	0.25	1.75	0.67	370.38	1.32	1.01	1.00	1.00	1.00	1.00
48	32	72	0.25	1.50	0.84	370.38	1.31	1.01	1.00	1.00	1.00	1.00
48	32	80	0.25	1.38	0.96	370.38	1.31	1.01	1.00	1.00	1.00	1.00
48	40	56	0.25	1.75	0.67	370.38	1.35	1.00	1.00	1.00	1.00	1.00
48	40	64	0.25	1.38	0.96	370.38	1.33	1.00	1.00	1.00	1.00	1.00
48	40	72	0.25	1.25	1.15	370.38	1.32	1.00	1.00	1.00	1.00	1.00
48	40	80	0.25	1.19	1.28	370.38	1.32	1.00	1.00	1.00	1.00	1.00

80 Streams; 16 Streams Off-target; s_0 = 64 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
64	8	72	0.25	6.25	0.16	370.38	53.79	5.54	1.73	1.23	1.11	1.07
64	8	80	0.25	3.63	0.28	370.38	51.65	4.99	1.68	1.26	1.16	1.12
64	16	72	0.25	5.50	0.18	370.38	53.89	5.54	1.72	1.21	1.09	1.05
64	16	80	0.25	3.25	0.32	370.38	51.86	4.99	1.66	1.23	1.12	1.08
64	24	72	0.25	4.75	0.21	370.38	54.00	5.55	1.71	1.20	1.08	1.04
64	24	80	0.25	2.88	0.37	370.38	52.13	5.01	1.65	1.21	1.10	1.06
64	32	72	0.25	4.00	0.25	370.38	54.15	5.56	1.70	1.19	1.07	1.03
64	32	80	0.25	2.50	0.43	370.38	52.49	5.05	1.64	1.19	1.08	1.04
64	40	72	0.25	3.25	0.32	370.38	54.38	5.60	1.70	1.19	1.06	1.02
64	40	80	0.25	2.13	0.52	370.38	53.03	5.13	1.63	1.18	1.07	1.03
64	48	72	0.25	2.50	0.43	370.38	54.82	5.68	1.71	1.18	1.06	1.02
64	48	80	0.25	1.75	0.67	370.38	53.99	5.30	1.65	1.18	1.06	1.03
64	56	72	0.25	1.75	0.67	370.38	56.03	5.96	1.74	1.19	1.06	1.02
64	56	80	0.25	1.38	0.96	370.38	56.24	5.80	1.70	1.18	1.06	1.02

80 Streams; 32 Streams Off-target; s_0 = 64 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
64	8	72	0.25	6.25	0.16	370.38	5.59	1.22	1.06	1.04	1.03	1.02
64	8	80	0.25	3.63	0.28	370.38	5.02	1.26	1.11	1.07	1.05	1.04
64	16	72	0.25	5.50	0.18	370.38	5.59	1.21	1.04	1.02	1.01	1.01
64	16	80	0.25	3.25	0.32	370.38	5.02	1.22	1.07	1.04	1.02	1.01
64	24	72	0.25	4.75	0.21	370.38	5.60	1.19	1.03	1.01	1.00	1.00
64	24	80	0.25	2.88	0.37	370.38	5.03	1.20	1.05	1.02	1.01	1.00
64	32	72	0.25	4.00	0.25	370.38	5.61	1.19	1.02	1.01	1.00	1.00
64	32	80	0.25	2.50	0.43	370.38	5.07	1.19	1.04	1.01	1.00	1.00
64	40	72	0.25	3.25	0.32	370.38	5.65	1.18	1.02	1.00	1.00	1.00
64	40	80	0.25	2.13	0.52	370.38	5.15	1.18	1.03	1.00	1.00	1.00
64	48	72	0.25	2.50	0.43	370.38	5.73	1.18	1.02	1.00	1.00	1.00
64	48	80	0.25	1.75	0.67	370.38	5.32	1.17	1.02	1.00	1.00	1.00
64	56	72	0.25	1.75	0.67	370.38	6.02	1.18	1.01	1.00	1.00	1.00
64	56	80	0.25	1.38	0.96	370.38	5.83	1.17	1.02	1.00	1.00	1.00

80 Streams; 48 Streams Off-target; s_0 = 64 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
64	8	72	0.25	6.25	0.16	370.38	1.73	1.06	1.03	1.02	1.01	1.00
64	8	80	0.25	3.63	0.28	370.38	1.69	1.11	1.05	1.03	1.01	1.01
64	16	72	0.25	5.50	0.18	370.38	1.71	1.04	1.01	1.00	1.00	1.00
64	16	80	0.25	3.25	0.32	370.38	1.66	1.06	1.02	1.01	1.00	1.00
64	24	72	0.25	4.75	0.21	370.38	1.71	1.03	1.00	1.00	1.00	1.00
64	24	80	0.25	2.88	0.37	370.38	1.64	1.05	1.01	1.00	1.00	1.00
64	32	72	0.25	4.00	0.25	370.38	1.70	1.02	1.00	1.00	1.00	1.00
64	32	80	0.25	2.50	0.43	370.38	1.63	1.03	1.00	1.00	1.00	1.00
64	40	72	0.25	3.25	0.32	370.38	1.70	1.02	1.00	1.00	1.00	1.00
64	40	80	0.25	2.13	0.52	370.38	1.63	1.02	1.00	1.00	1.00	1.00
64	48	72	0.25	2.50	0.43	370.38	1.70	1.01	1.00	1.00	1.00	1.00
64	48	80	0.25	1.75	0.67	370.38	1.64	1.02	1.00	1.00	1.00	1.00
64	56	72	0.25	1.75	0.67	370.38	1.74	1.01	1.00	1.00	1.00	1.00
64	56	80	0.25	1.38	0.96	370.38	1.70	1.01	1.00	1.00	1.00	1.00

80 Streams; 64 Streams Off-target; s_0 = 64 Streams

s_0	s_1	s_2	t_1	t_2	w	0.0	0.5	1.0	1.5	2.0	2.5	3.0
64	8	72	0.25	6.25	0.16	370.38	1.22	1.03	1.01	1.00	1.00	1.00
64	8	80	0.25	3.63	0.28	370.38	1.26	1.06	1.02	1.01	1.00	1.00
64	16	72	0.25	5.50	0.18	370.38	1.20	1.02	1.00	1.00	1.00	1.00
64	16	80	0.25	3.25	0.32	370.38	1.22	1.03	1.00	1.00	1.00	1.00
64	24	72	0.25	4.75	0.21	370.38	1.19	1.01	1.00	1.00	1.00	1.00
64	24	80	0.25	2.88	0.37	370.38	1.20	1.01	1.00	1.00	1.00	1.00
64	32	72	0.25	4.00	0.25	370.38	1.18	1.00	1.00	1.00	1.00	1.00
64	32	80	0.25	2.50	0.43	370.38	1.18	1.01	1.00	1.00	1.00	1.00
64	40	72	0.25	3.25	0.32	370.38	1.18	1.00	1.00	1.00	1.00	1.00
64	40	80	0.25	2.13	0.52	370.38	1.17	1.00	1.00	1.00	1.00	1.00
64	48	72	0.25	2.50	0.43	370.38	1.18	1.00	1.00	1.00	1.00	1.00
64	48	80	0.25	1.75	0.67	370.38	1.17	1.00	1.00	1.00	1.00	1.00
64	56	72	0.25	1.75	0.67	370.38	1.18	1.00	1.00	1.00	1.00	1.00
64	56	80	0.25	1.38	0.96	370.38	1.17	1.00	1.00	1.00	1.00	1.00

CHAPTER 5

MULTIPLE STREAM FILLING OPERATIONS: A CASE STUDY

Introduction

In order to demonstrate the concepts of the previous chapters, and in the interest of applying the theoretical results to an actual process, a case study is investigated. The process considered in this chapter, is an actual manufacturing operation where data and information has been gathered directly from the system in its present state. The implementation of the concepts described in this chapter is hypothetical as the actual process is not currently able to implement all suggestions.

Most of the assignable causes known at the facility impact all streams on a single machine. Single stream issues also occur but are usually related to the failure of specific valves. Situations causing subsets of valves to shift off-target could arise if parts used in rebuilding the valves share a common faulty source (e.g. weak springs, etc.).

One of the issues generating the most concern is fill performance at machine start-up when the line switches to a new product. The fill valves are not optimized for any particular product and it is suspected that while the machines have been adjusted to perform adequately, several streams may be off-target and could be tuned at the start of a new product run to improve product yield.

Rapid detection of process shifts is desired as production runs are relatively short. Small process shifts noticed near the end of a production run are not worth correcting as the loss of production will greatly outweigh any potential gain of product yield at that

point. For this reason shifts which impact a large number of the valves are more important to detect than small shifts impacting only a few valves.

Background

The process considered is a high-speed beverage filling operation of a local beverage bottler. The process consists of several "bottling lines" with high-speed filling machines for cans (12 oz.), and plastic bottles (12 oz., 20 oz., 500 ml., 1000 ml., and 2000 ml.). Table 5-1 shows typical product lines including the size of the can or bottle being filled, the number of valves on each filler, and the rate at which the filling machines operate under normal conditions. The product flavor filled on each line depends on current demand and varies from shift to shift. Each shift is 8 hours long and filling operations continue 24 hours a day, 7 days a week, although Sunday is usually reserved for routine equipment maintenance.

TABLE 5-1. Production Information for a Typical High-Speed Bottling Operation

Line Number	*Product*	*Number of Valves*	*Rate: product/min*
1	12 oz cans	120	1600
2	12 oz cans	72	1200
3	500 ml bottles	72	600
4	1000 ml bottles	60	300
5	2000 ml bottles	52	260

The monitored response of interest in this study is fill volume as determined by fill height and/or fill weight (tare weight). The tare weight is dependent on the product being filled. Under-filling of product is a concern because of truth in advertising

requirements. For example, if a can says 12 ounces on the label, there needs to be 12 ounces in the can. If the fill average of a 12-ounce product falls below 11.9 ounces, the line will shut down. If any individual can is measured below 11.7 ounces it will be scrapped. Over-filling is a concern because of lost revenue. For example, it is estimated for the high-speed can lines that 1/20th of an ounce (0.05 oz.) of overfill costs the company about $20,000 a month.

Before discussing the specifics involved in measuring fill volume, it is instructive to have a basic understanding of the entire bottling process. Primary elements of a typical filling operation are shown in Figure 5-1. Syrup is delivered to the company from a single outside supplier. This syrup is stored in one of 9 syrup storage tanks depending on flavor. Only single flavors are run on any given filling machine at one time. Usually a single flavor is produced during a shift, although occasionally a flavor change will be made during a shift.

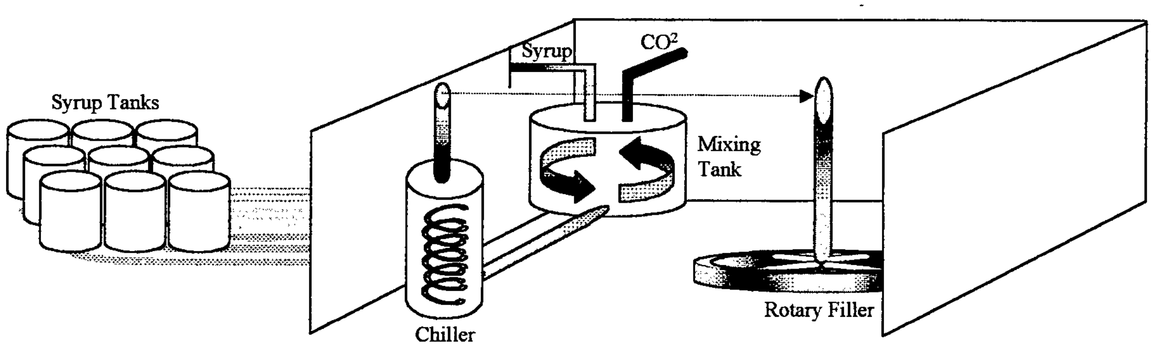


FIGURE 5-1. Typical Filling Operation Elements

From the storage tanks, the syrup is mixed with carbonated water. As the product is now carbonated, special handling is required. The product is chilled to reduce the amount of carbon dioxide (CO_2) evaporation. Carbonated liquids cannot be pumped throughout the facility, therefore the product is moved under pressure of 50-60 pounds per square inch (psi) during the bottling operation. The pressure also helps further reduce CO_2 evaporation. There is a single primary delivery pipe from the chillers to each bottling machine. At the filler, the pipe splits off into 4 secondary pipes that fill a common bowl from which all valves draw.

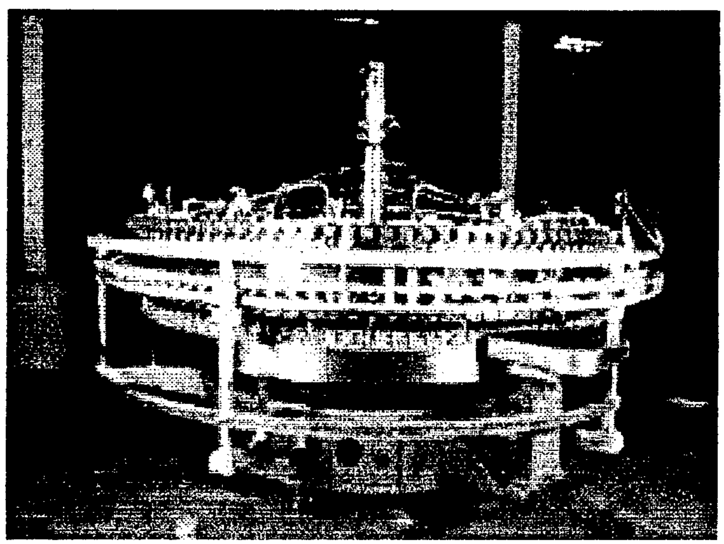


FIGURE 5-2. Rotary-type Filling Machine

A representative, rotary-type, filling machine is shown in Figure 5-2. A rotary filling system has fill valves arranged on a carousel that collects cans or bottles from a conveyer. The entire bowl and valve assembly rotates about a central axis during the

filling cycle. As empty containers enter the filling machine they are married up with a fill valve and the container and valve rotate together during the filling process. The fill usually takes about 200° of revolution, or just over one-half of a rotation, although the can/bottle remains with the machine for about 320° of rotation. After filling, the full container is released from the rotary filler and passes down a short conveyor to a "capping" machine. The product then proceeds down the line to be packaged, stored, and shipped.

While filling requires only a short amount of time, it is an important and complicated procedure. A brief description of the process will help clarify some of the assignable causes associated with filling problems. For purposes of this discussion, we will only consider bottle-filling operations, although can filling is very similar. Numbers used in this description refer to the valve cutaway drawing in Figure 5-3.

When a bottle enters the fill machine it is seated in a rubber seal (3) and pressurized. This counter-pressure equalizes the pressure in the bottle with that in the fill tank to prevent foaming. The valve is opened when the cam (1) strikes a plunger that only deploys if a bottle is present. The liquid enters the bottle through the fill nozzles (6) forcing the liquid to the sides of the bottle and filling occurs from the bottom, up. Since the bottle and the liquid are under the same pressure, no filling occurs until pressure is allowed to escape through the counter-pressure sleeve(4).

Filling continues until the ball check (5) is carried by the volume of the liquid to the opening of the counter-pressure sleeve stopping the release of pressure and as a result, stopping the fill. As the machine rotates, the valve is then closed via the cam. Before the

bottle comes off the filler, a tapered block makes contact with the snift button (2) allowing a controlled release of pressure to prevent foaming. The bottle then passes over a short bridge to the capping machine where the product is sealed by screwing on twist-off caps.

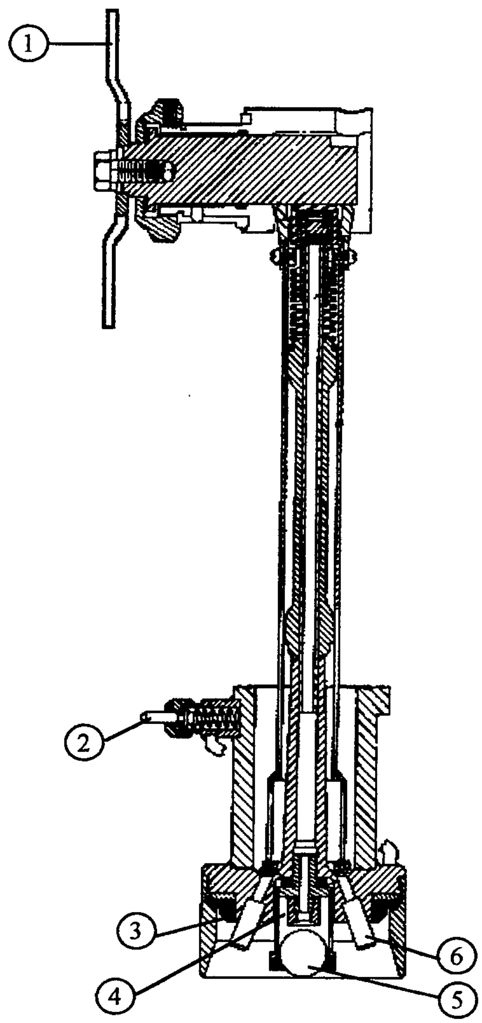


FIGURE 5-3. Uniblend Filling Valve

Product measurement is accomplished in four ways. The first is a visual inspection by the fill machine operator who watches the bottles as they pass over the bridge between the filler and the capper. The operator watches for the color changes that indicate foaming problems, identifies pressure problems that cause the beverage spurt out of the bottle with a geyser-like effect, and monitors fill height by sight.

Here the description of bottling and canning operations diverges. Since the plastic bottle is transparent, the consumer can monitor fill height after a fashion while selecting which bottle to purchase in the supermarket. In the canning process the lids are sealed onto the cans using a crowning machine. After crowning, the cans pass through a fill-detection machine which uses gamma rays to identify any can filled below 11.7 ounces. Low filled cans are literally "kicked out" of the system and scrapped. This low fill detection method for canned products is not available for bottled products. The scrap bins that collect the kicked out cans provide some measure of fill performance as inordinate amounts of cans in the bin suggest a related problem with the filling machine. The bottling lines require this information to be obtained via sampling methods.

All lines in the process are controlled/monitored by various means. Engineering process control (EPC) plays a large part in the control process. Levels of CO_2, sweetener or BRIX, pressure, and temperature are all continuously adjusted to maintain an ideal mix. In addition, samples are also taken from each line. Each fill machine is sampled five minutes after process start-up, and then again once per hour until the product on that line changes. The samples are weighed using tare weight to determine fill accuracy as

well as being tested for carbonation, BRIX content, taste and the seal of the can lid or bottle cap.

Most filling issues concern assignable causes which impact all, or most of the fill valves. A new issue at the facility has to do with the product being bottled on a particular line. Recently the facility has begun bottling fruit juices containing pulp. Figure 5-4 shows how the product is transported to the bowl from which each valve draws while filling. Pulp concentrations tend to vary from where each feeder line joins the bowl to the areas situated directly between feeder lines. The pulp can begin to clog valves and restrict product flow during filling operations.

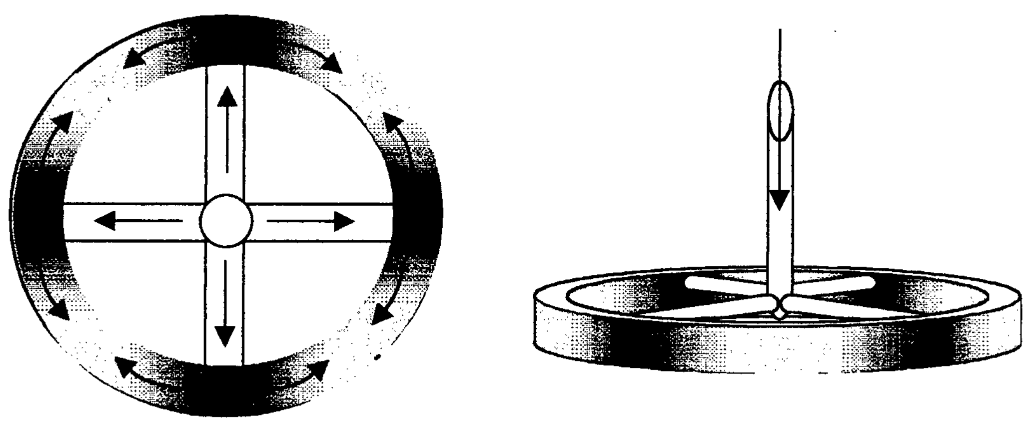


FIGURE 5-4. Product Distribution to Filling Bowl and Pulp Concentrations
(Darker shaded areas indicate more pulp)

Current Situation

For the purposes of this study we will consider line 5, the two-liter bottling line. This line has a rotary-filling machine with 52 valves and a normal operating speed of about 260 bottles filled per minute. One of the methods employed to ensure product quality is to measure the fill volume of bottles sampled from the line using tare weight. At present 5 bottles are sampled from the line 5 minutes into each flavor run and then samples of 5 bottles are drawn once an hour for the remainder of the flavor run.

The current monitoring approach assumes any assignable cause impacts all valves equally and all streams are independent and identically distributed following an approximate normal distribution. The target value for each stream is 2000 milliliters (ml.) and each stream is known to have a constant standard deviation of 10 ml. when the process is on-target. Under these assumptions, the system is thought to be monitored with the protection shown in Table 5-2.

TABLE 5-2. ARL Values for an $\overline{X}$ Chart with n = 5 Bottles

δ *Shift*	*ARL*
0.0	370.38
0.5	33.40
1.0	4.50
1.5	1.57
2.0	1.08
2.5	1.00
3.0	1.00

Using Table 5-2 and knowing that assignable causes are desired to be detected early in the process operation, it can be assumed that shifts of 1.5σ and greater are

generally noticed in time to adjust the process. Meanwhile, shifts of 1.0σ and less are not caught in time to do anything about them. Recall that the ARL is a measure of the number of samples taken before the chart signals. Since the first sample occurs 5 minutes after process start up, an ARL of 1.57 equates to about 0.65 hours of operation before a chart signals, on average. Naturally, since samples are taken at hourly intervals after the 5-minute check, the signals actually occur at either the 5-minute check or at the first hourly sample for shifts of 1.5σ.

Since this is a multiple stream process and it is suspected that some assignable causes are impacting several, but not all process streams, the actual protection afforded with the current sampling plan is somewhat different. Table 5-3 shows the actual protection obtained by monitoring the multiple stream process with the current sampling plan. The last row shows how the $\overline{X}$ chart will behave if all valves shift and corresponds to the values given earlier in Table 5-2. Recall that the ARL measures the average number of plotted points until a chart signals. Since the first sample is taken 5 minutes into the production run, the ARL values can be converted to ATS values by subtracting 55 minutes, or 0.92 hours from the ARL values in Table 5-2. Notice under these conditions, if half the valves shift by 1.5σ the chart will not signal until the 7.03 samples have been taken on average. Converting this to an ATS works out to about 6.11 hours of production time.

TABLE 5-3. ARL Values for Sampling 5 bottles from 52 Streams

# Streams Off-target	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*
1	361.60	335.42	293.58	241.58	188.00	140.52
2	351.77	300.17	229.98	161.70	108.88	73.24
5	317.64	206.64	113.88	61.24	35.11	22.06
10	255.06	109.20	44.17	20.66	11.54	7.48
13	219.49	76.65	28.09	12.88	7.33	4.90
26	110.25	22.23	7.03	3.42	2.23	1.73
39	58.30	9.04	2.90	1.63	1.26	1.12
52	33.40	4.50	1.57	1.08	1.00	1.00

To put things in perspective, consider how much waste would be associated with a shift above target. With a standard deviation of 10 ml. and operating speeds of 260 bottles per minute, a shift of 1.5σ on half the valves will result in an average waste equaling over 350 two-liter bottles before generating a signal on a monitoring chart. This works out to nearly 1 bottle a minute for over 6 hours of operating time.

Alternative Monitoring Schemes

Increased Sample Size. A quick way to improve the monitoring performance of the process is evident from watching the way in which samples are collected. In addition to the 5 bottles used for fill height sampling, 9 other bottles are also pulled from the line at the same time for a total of 14 bottles. Destructive testing requires 2 of the bottles, 1 for carbonation testing and the other for BRIX testing. The remaining 12 bottles are needed for testing the seal of the bottle capping machine. Figure 5-6 shows the bottling operation to include the capping machine. This capping machine is made up of 12 heads which screw caps on the bottles using a small rotary-type

machine. Taking samples of size 14 enables the analyst to conduct the necessary destructive tests and test all 12 capping heads using a torque test.

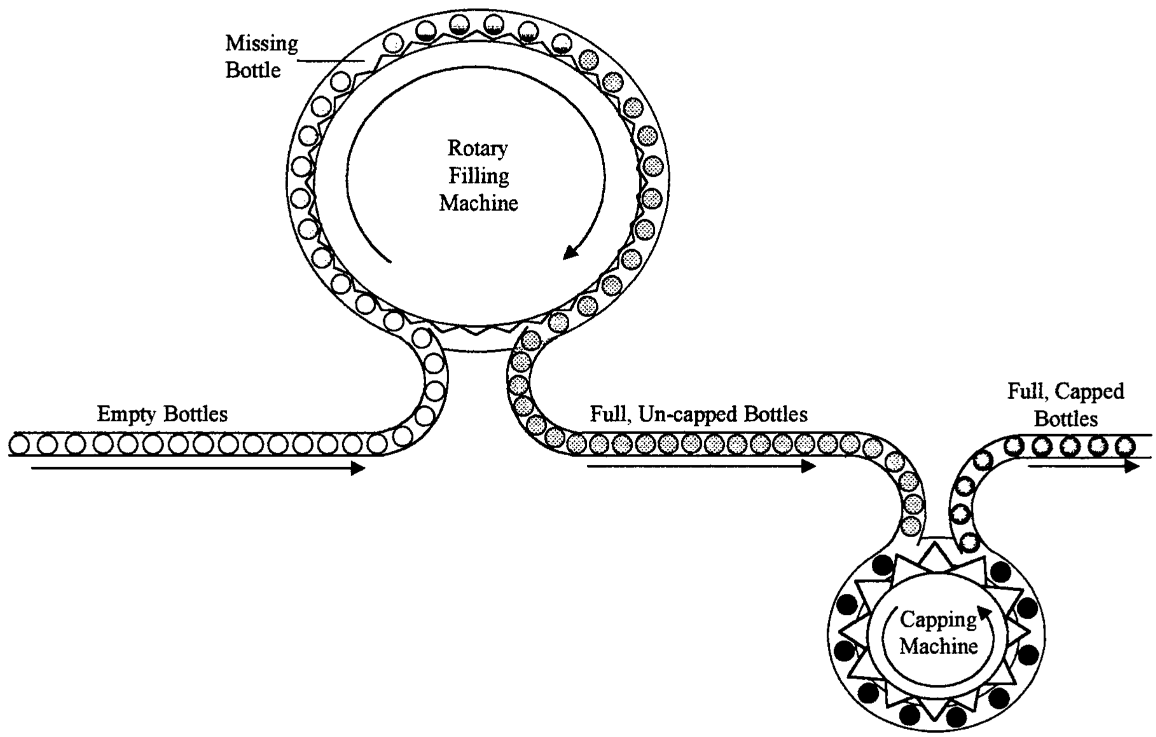


FIGURE 5-6. Rotary Filling and Capping Processes

By taking advantage of these extra bottles, it should be possible to sample the fill height of at least 13 bottles during each sample time. This number was chosen as it conveniently represents 25% of the total valves and allows for 1 of the destructive tests to begin immediately if necessary. (The second destructive test would merely have to wait

until 1 bottle was measured using tare weight, a process that takes less than 10 seconds.) Table 5-4 shows average run length results if 13 bottles are used at each sample point.

TABLE 5-4. ARL Values for Sampling 13 bottles from 52 Streams

# *Off-target*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
1	361.73	337.21	300.73	257.64	213.35
2	349.51	295.50	227.88	164.61	114.70
5	297.97	173.64	89.22	46.20	25.68
10	201.50	66.57	23.98	10.72	5.91
13	154.09	39.69	13.05	5.88	3.42
26	49.40	7.33	2.49	1.48	1.18
39	18.98	2.55	1.22	1.03	1.00
52	8.65	1.37	1.01	1.00	1.00

Notice under these new conditions, if half the valves shift by 1.5σ, the chart will, on average, signal before the second hour (2.49 ARL $\cong$ 1.57 hours ATS). Using the previous example, again consider the waste associated with a shift above target of 1.5σ on half the valves. This time the waste equals just over 91 two-liter bottles before a signal is generated.

Adaptive Alternatives. To further improve the process monitoring performance a fractional sample adaptive approach is implemented. Selecting a reasonable large sample size, s_2, will be an important consideration. In observing various analysts collect samples it was noted that some analysts would take 12 bottles for the cap-seal torque test, plus 2 bottles for destructive testing, plus 5 additional bottles for fill height monitoring. Since 17 bottles were being collected using this particular approach, an upper sample size limit of 20 bottles seems reasonable. To keep the larger sample size

an integer multiple of the smaller, s_1 was established at 10 bottles. Letting $s_0 = 13$ bottles for comparison with the previous method and choosing $t_1 = 0.25$ hours, or 15 minutes, established a threshold value and upper sampling interval of $w = 1.03$, and $t_2 = 1.32$ hours respectively. The ARL results for this monitoring scheme are shown in Table 5-5.

TABLE 5-5. ATS Values for VSSI Sampling 52 Streams
Using $s_1 = 10$, $s_2 = 20$, $s_0 = 13$, $t_1 = 0.25$, $t_2 = 1.32$, and $w = 1.03$

# *Off-target*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
1	361.29	335.51	297.16	252.02	206.02
2	348.47	291.87	221.54	156.72	106.69
5	294.29	164.57	79.13	37.87	19.47
10	192.20	54.06	15.81	6.08	3.23
13	141.82	28.05	7.10	2.98	1.90
26	34.07	3.06	1.41	1.15	1.07
39	8.74	1.37	1.07	1.02	1.00
52	3.11	1.10	1.01	1.00	1.00

It is important to remember that the VSSI results are reported in terms of average time to signal. Using the adaptive approach when half the valves shift by 1.5σ results in a signal after 1.41 hours of operation on average. Using the same example as before (1/2 the valves shifting above target by 1.5σ), the waste only decreases a modest amount to about 82 two-liter bottles before a signal is generated. This seems like a small gain for a rather involved change in operating procedures; however, the ATS values in Table 5-5 are computed assuming the shift occurs at a random point in time. If the off-target valves are assumed to be present at process start-up, the values in Table 5-5 should be adjusted by subtracting 0.17 hours. For the previous example the signal time be reduced to 1.24 hours resulting in a waste of about 72 bottles – a slight improvement.

An even larger improvement can be made if we are going to assume an off-target condition present at start up. Recall that the **b′** vector used to determine the ATS uses values of b_1 and b_2 that represent the average proportion of time spent in zones 1 and 2 when the process is under control. Dramatically different results are obtained if the process is assumed to be in zone 2 when the shift occurs. The reason for this is that the adaptive scheme now has a "head start" toward catching the off-target condition. This is why the fast initial response (FIR) is recommended at start-up. Since off-target conditions present at start-up are known to be an issue and should be detected quickly to enable the maximum improvement in process yield, it is worth considering how the adaptive scheme performs using the FIR option when the process is off-target at process start-up. To generate ATS values in this situation, change the initial probabilities to of $b_1 = 0$, and $b_2 = 1$. Table 5-6 shows the FIR ATS results assuming some valves are off-target at start-up and the first sample occurs 5 minutes into production.

TABLE 5-6. FIR ATS Values for VSSI Sampling 52 Streams Using $s_1 = 10$, $s_2 = 20$, $s_0 = 13$, $t_1 = 0.25$, $t_2 = 1.32$, and $w = 1.03$

# *Off-target*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
1	360.37	334.59	296.24	251.12	205.14
2	347.54	290.93	220.59	155.77	105.74
5	293.31	163.50	77.99	36.71	18.29
10	191.08	52.77	14.49	4.78	1.97
13	140.62	26.68	5.76	1.71	0.70
26	32.63	1.76	0.28	0.12	0.09
39	7.31	0.26	0.09	0.08	0.08
52	1.79	0.10	0.08	0.08	0.08

Now if half the valves are off-target by 15 ml. (1.5σ) at start-up, the adaptive scheme will generate a signal, on average, in 17 minutes (0.28 hours). This amounts to only about 16 bottles of waste. This is a 95 percent reduction in waste from the original 5-bottle sample scheme and an 82 percent reduction from the 13 bottle, fixed sample plan, for this scenario. Furthermore, notice from Table 5-6 that when all the streams shift by any size larger than 0.5σ, the adaptive scheme usually signals during the 5-minute check. This provides an excellent opportunity to make necessary corrections early and have the system running with minimal waste (maximum yield) for most of the flavor run. Finally, note that with a threshold value of 1.03, the adaptive monitoring scheme will operate in zone 1 approximately 70% of the time while the process is on-target. This means most samples for an on-target process will be of size $s_1 = 10$, 1.32 hours apart.

Summary

This case study shows that a process with a large number of streams can be effectively monitored by sampling only a fraction of the total streams. For the specific process studied in this chapter, a minimum recommendation would be to measure the fill height for as many bottles as possible. If 14 bottles are being pulled from the production line every hour, the fill height sample size should not be limited to only a 5-bottle sample. Furthermore, if the process is flexible enough to allow it, an adaptive approach enabling the advantages of the FIR technique ought to be implement.

CHAPTER 6

SUMMARY AND CONCLUSIONS

Contributions

This research builds on the work presented in the literature for moderate numbers of streams, average run length determination, adaptive monitoring methods, and associated techniques for determining adaptive chart performance. Original contributions are produced for multiple stream processes with large numbers of streams where it is possible to monitor only a fraction of the total streams at a given time. This is the first presentation of issues surrounding fractionally sampled multiple stream processes. This situation is of interest in those processes where the speed of production is great and includes a large number of streams, but the ability to monitor the process is not fully automated and unable to keep up with the speed of production.

A probability model for determining detection probabilities in fractionally sampled multiple stream processes was developed. This model measures the likelihood that an $\overline{X}$ chart for a fractionally sampled system will signal an off-target condition when any fraction of the streams shift. In addition to the mathematics involved in computing this detection probability, a computer program was given which automates the process and quickly gives a result for a process with any number of streams and allows an infinite number of combinations of stream shift scenarios to be examined. Results from several of these scenarios have been tabulated and graphed.

Adaptive approaches to system monitoring were applied to multiple stream processes in general and the fractional sampling problem specifically. While a recent, thorough survey of available adaptive techniques is presented by Tagaras (1998), this study represents the first integration of adaptive techniques and multiple stream processes. Another original contribution of this study is the construction of a Markov chain method that incorporates the new probability model to measure the performance of adaptive schemes of monitoring fractionally sampled multiple stream processes. This procedure relies on the probability of detection algorithm presented earlier to establish the transition probability matrix. The ATS results were used to identify promising adaptive sampling schemes for monitoring a MSP using fractional samples. It was shown that the adaptive fraction approach gave superior results to the fixed fraction scheme and often yielded results nearly as good as those obtained by sampling all the streams involved in a process.

Finally an in depth example was provided by means of a case study where the methods described in this study were applied.

Suggestions for Additional Research

Avenues for future research addressing the issues of multiple stream processes abound. This area will remain diverse as multiple stream issues continue to involve processes with small to moderate numbers of streams and increasingly large numbers of streams and high rates of production. As the rates of production skyrocket, the need for low false alarm rates will be increasingly important, while at the same time, system shifts

will need to be signaled faster than ever. Automated monitoring methods introduce their own problems in both data correlation and sheer volume of available data.

One interesting problem involves how to monitor a MSP when the streams arise from differing distributions. An example might be where the streams near the edge, that is the first few streams and the last few streams, behave differently than the middle streams. This situation might occur in a web-type process such as in paper production, or tape manufacture. Figure 6-1 shows a cross section of the streams in such a system and how the edges appear different than the center. One possible approach might be to develop a multiple stream charting procedure based on model-free techniques. This approach seems a natural fit for any process complicated by streams from varying distributions. Furthermore, this technique has been used to successfully address autocorrelation issues in the univariate case.

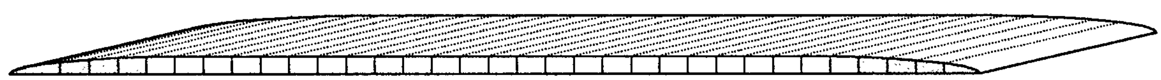


FIGURE 6-1. Web Process with Different Distributions for Middle- and End-Streams

Correlation within multiple stream processes is an issue that needs attention. The ability of some processes to monitor all items produced has led to correlated data situations where data independence had been previously assumed. Application of current techniques for dealing with correlated data in single stream and multivariate situations might be pursued. One possible approach would be to represent the process using a time

series model and chart the residuals. In a multiple stream environment, a time series model might be required for each stream and then apply existing MSP techniques to the residuals. Perhaps a single value, such as the maximum value across all streams, could be modeled in a time series fashion. Another approach might consider the streams in a p stream process as months within a p-month long year and use a seasonal time series model to monitor the system.

Statistical pattern recognition algorithms might be developed. The very large number of multiple streams problem may benefit by allocating each stream to a "bandwidth" and setting a desired pattern for the process as a whole. In the bottling example, if the desired fill height across all containers is equal, say 12 ounces, then the target distribution across all streams at a given time would be the uniform distribution. Deviations from the target pattern could be detected by the failure to fit a recognized pattern within statistical tolerances. Allowances for non-identical streams might incorporate maximum flexibility into allowable pattern definitions. In this case, the problem of massive data sets becomes a benefit as they allow more nearly continuous pattern monitoring.

While this approach is probably the most complex, it offers some interesting possibilities. For example, genetic algorithms might be used to generate new off-target templates thereby allowing the monitoring process to detect new assignable causes. Specific patterns could also be established for known assignable causes and then a control chart signal would also immediately narrow down the interpretation issue. Even if a full pattern recognition scheme is not implemented, the concept of artificial

intelligence (AI) might be exploited. Patrick and Fattu (1986) define AI saying rather than providing help in generating decision rules, artificial intelligence instead provides procedures for interpreting given patterns and general bookkeeping strategies. With high-speed automated processes, it seems natural to try and implement some form of automated control chart scheme.

Conclusion

This study has shown that multiple stream processes can be effectively monitored when only a fraction of the total streams are sampled at a given time. Performance measures have been presented to help determine the risk associated with a fractional sampling scheme and a technique for applying adaptive sampling methods have been given. These results show that several alternative approaches to sampling only a portion of the total streams are available. In addition, some processes that are able to monitor all streams, may be able to benefit by occasionally using a fractional sample along with full samples. Furthermore, for systems that are able to withstand minor shifts and desire to catch moderate, or larger shifts, a fractional approach may present some new monitoring alternatives. Finally, any process which is upsizing need not fear outgrowing their ability to sample from all streams as fractional sampling methods could be used until such time as they may decide to also upsize their sampling ability.

While several new ideas have been presented for the multiple stream problem, they are by no means exhaustive. The earlier discussion of potential future research

shows a veritable cornucopia of opportunities are available for further study in the large number of streams problem as well as the multiple stream process problem in general.

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BIOGRAPHICAL SKETCH

Captain Jeffrey Wayne Lanning was born in Holland, Michigan, on the 14th of November 1964. He received his elementary education at Childs Elementary School in Bloomington, Indiana. His secondary education was completed at Laramie Senior High School in Laramie, Wyoming in 1983. After high school he attended the University of Wyoming for one year before being accepted to the U.S. Air Force Academy. He graduated with military distinction from the Air Force Academy in June of 1988 holding a Bachelor of Science degree and received a regular commission in the U.S. Air Force. His first assignment with the Air Force involved three years on a classified test program where he served as the lead human factors analyst. In March of 1993 Captain Lanning completed a Master of Science degree in operations research at the Air Force Institute of Technology. Following graduation he moved on to the Headquarters Air Force Operational Test and Evaluation Center for two years as an effectiveness analyst and program manager and became the Center's focal point for operational testing of the global positioning system. In August 1995 Captain Lanning entered the Graduate College at Arizona State University to pursue a doctorate in Industrial Engineering with an emphasis on quality and reliability engineering. Following graduation Captain Lanning will take a position as a professor at the Air Force Institute of Technology. He is a member of the Omega Rho and Alpha Pi Mu honor societies as well as a member of the American Society for Quality Control. Captain Lanning is married to the former Anita Lynn Haines. They have no children, but do have a dog and some houseplants.